Lecture 10: Synthesis of Heat Exchanger Networks

Chapter 16 of text-book - the concepts and methods highlighted through examples 16.1 to 16.4.

(With input from Dr. Xiaodong Liang)

Heat Exchanger Network Synthesis (HENS) Problem

Given

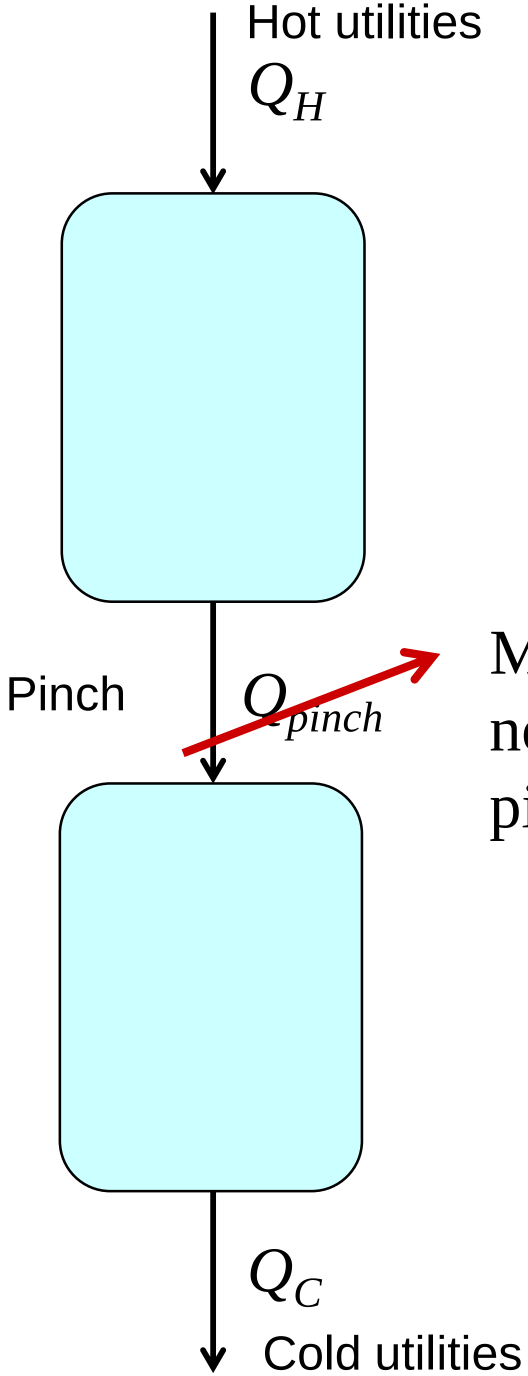
- **A set of hot process streams that needs to be cooled and a set of cold process streams that needs to be heated**
- **The flowrates and the inlet and outlet temperatures for all these process streams**
- **The heat capacities for each of the streams versus their temperatures as they pass through the heat exchange process**
- **The available utilities, their temperatures, and their costs per unit of heat provided or removed**

Determine

The heat exchange network for energy recovery that will minimize the annualized cost of the equipment plus the annual cost of utilities

Summary of the methods from Chapter 10

- **Given a minimum temperature approach (ΔT), the exact amount for minimum utility consumption can be predicted prior to developing the network structure**
- **Based on the pinch temperatures for minimum utility consumption, the synthesis of the network can be decomposed into sub-networks**
- **The fewest number of units in each sub-network is often equal to the numbr of process and utility streams minus one**
- **It is possible to develop good a priori estimates of the minimum total area of heat exchange in a network**



Splits the problem into two sub-networks: One above the pinch (cold uti;ities cannot be used) …

Minimum utility dictates that no heat is passed across the pinch

… also, one below the pinch (hot utilities cannot be used here)

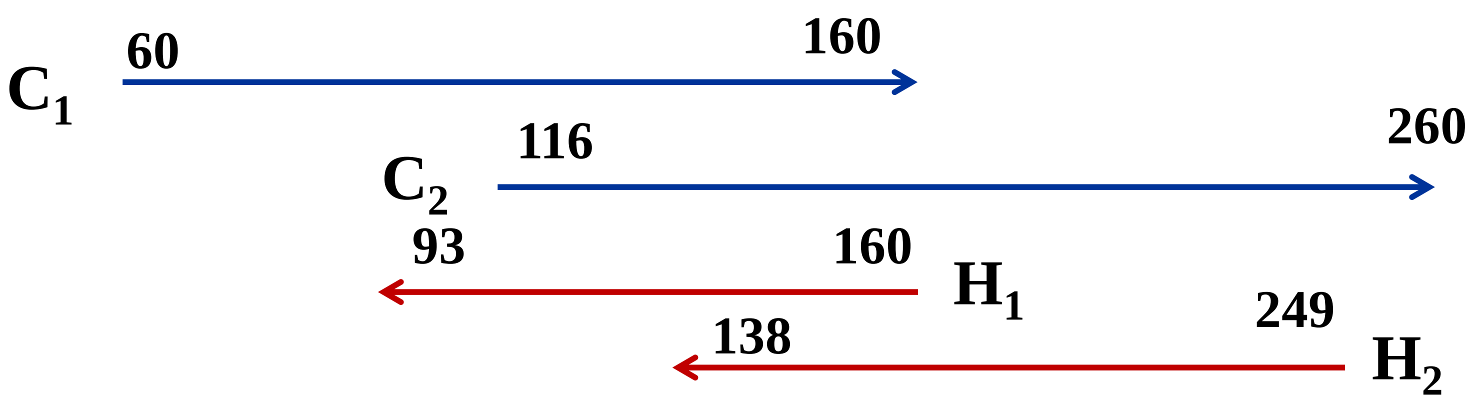
Heat Exchanger Network Synthesis (HENS) Problem

Given

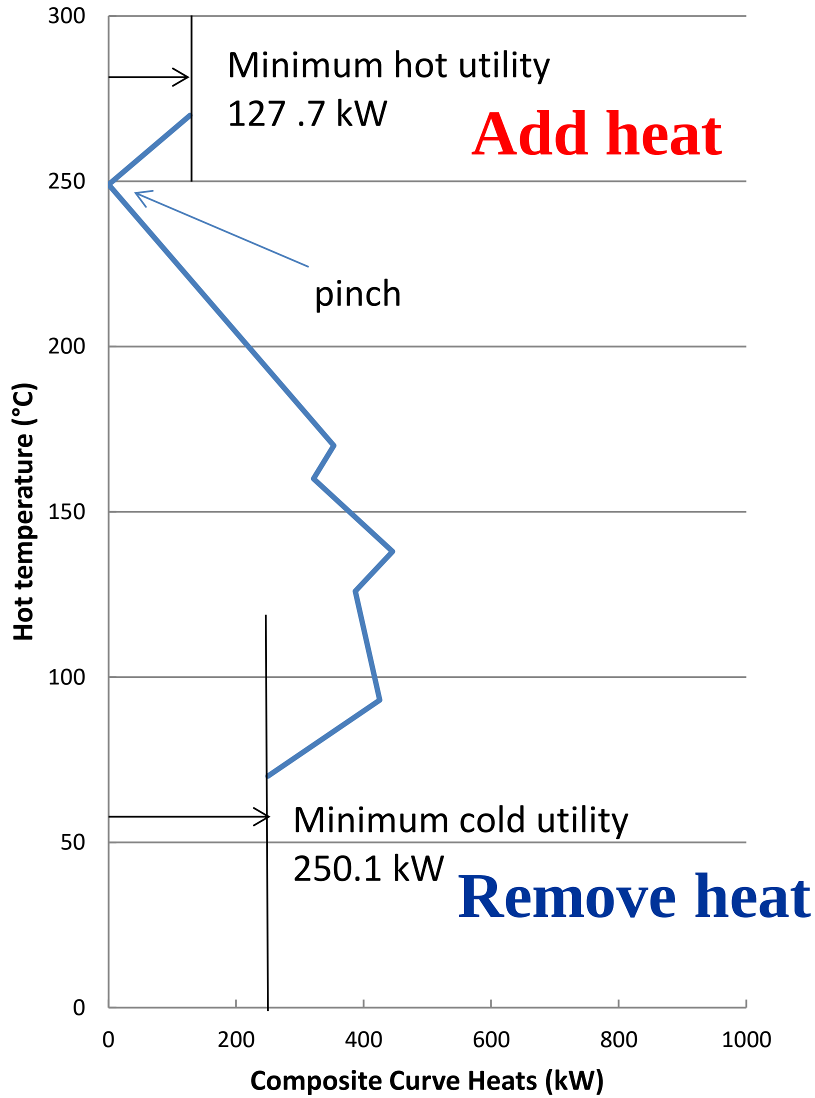
Stream	FC_p, kW/°C	T_{in}, °C	T_{out}, °C	Heat flow out, kW
C1	7.62	60	160	–762.0
C2	6.08	116	260	–875.5
H1	8.79	160	93	588.9
H2	10.55	249	138	1171.1

Determine

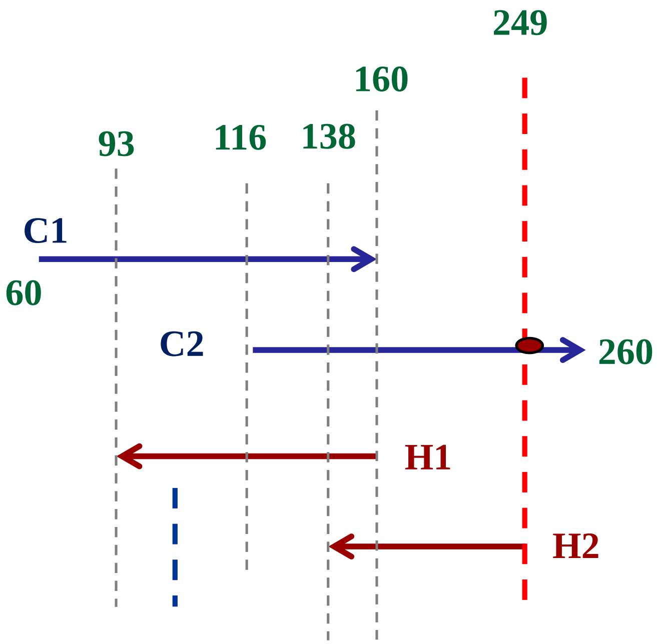
The heat exchange network for energy recovery that will minimize the annualized cost of the equipment plus the annual cost of utilities



Heat Exchanger Network Synthesis (HENS) Problem



One network above pinch
one network below pinch



Heat Exchanger Network Synthesis (HENS) Problem

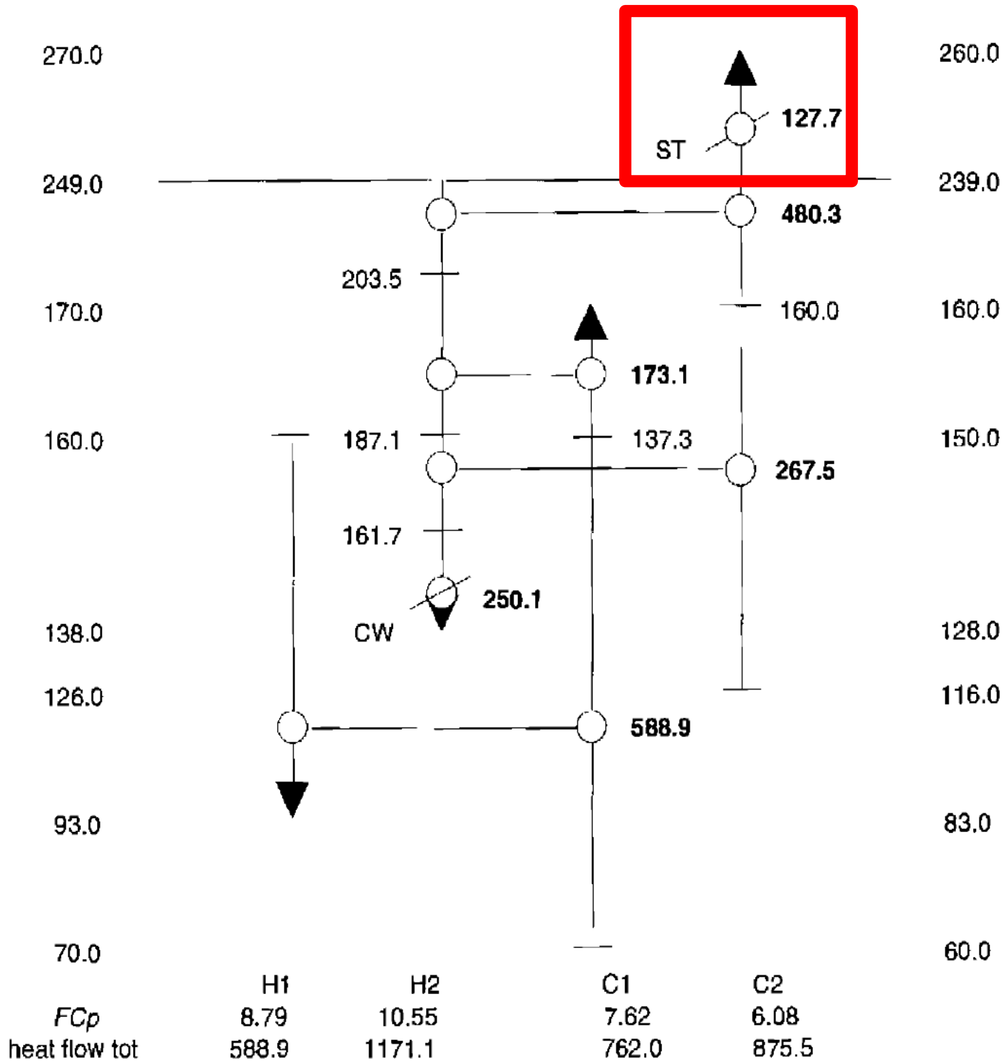
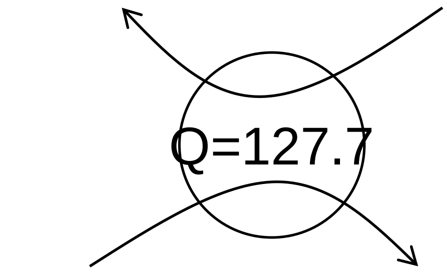


FIGURE 10.15 A possible heat exchanger network for 4SP1.

Above pinch only C2 and steam exist, therefore only one exchanger needed



Heat Exchanger Network Synthesis (HENS) Problem

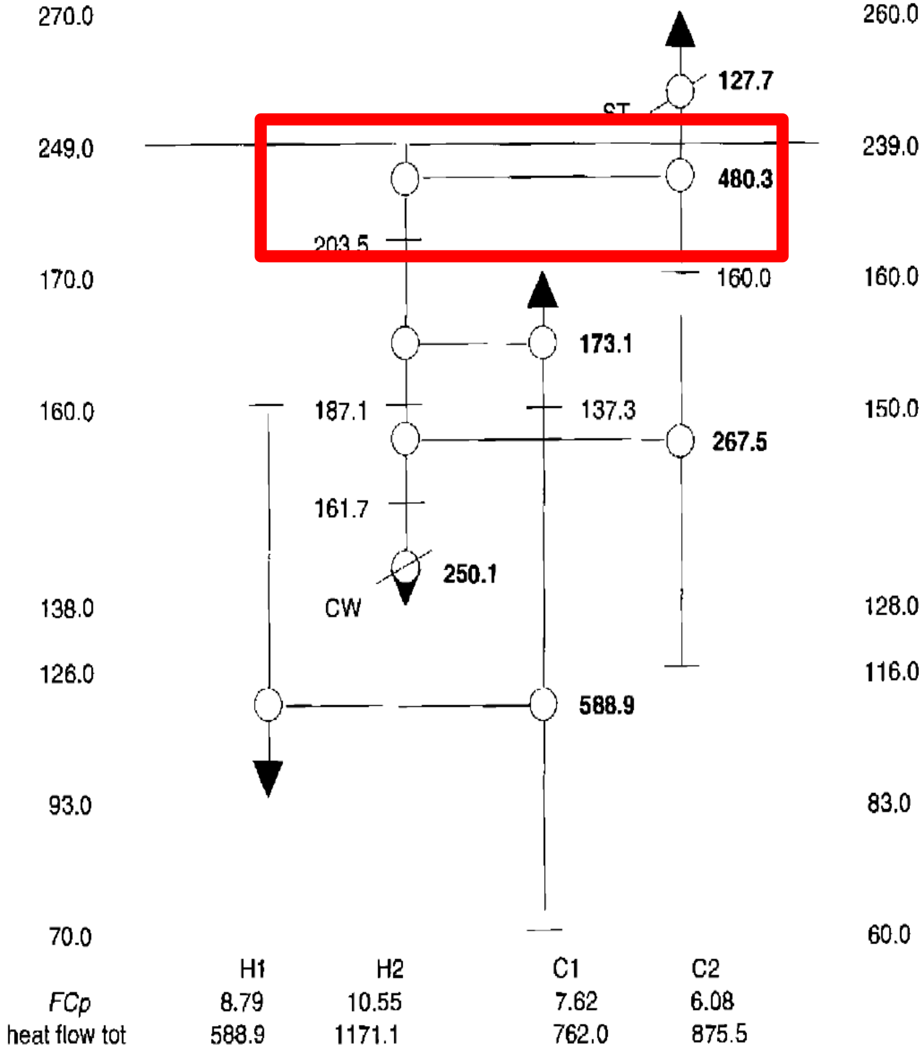
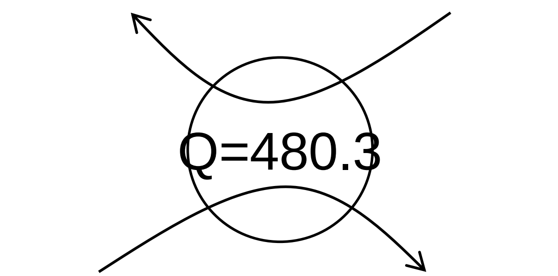


FIGURE 10.15 A possible heat exchanger network for 4SP1.

Exchange C2 and H2 below pinch



Heat Exchanger Network Synthesis (HENS) Problem

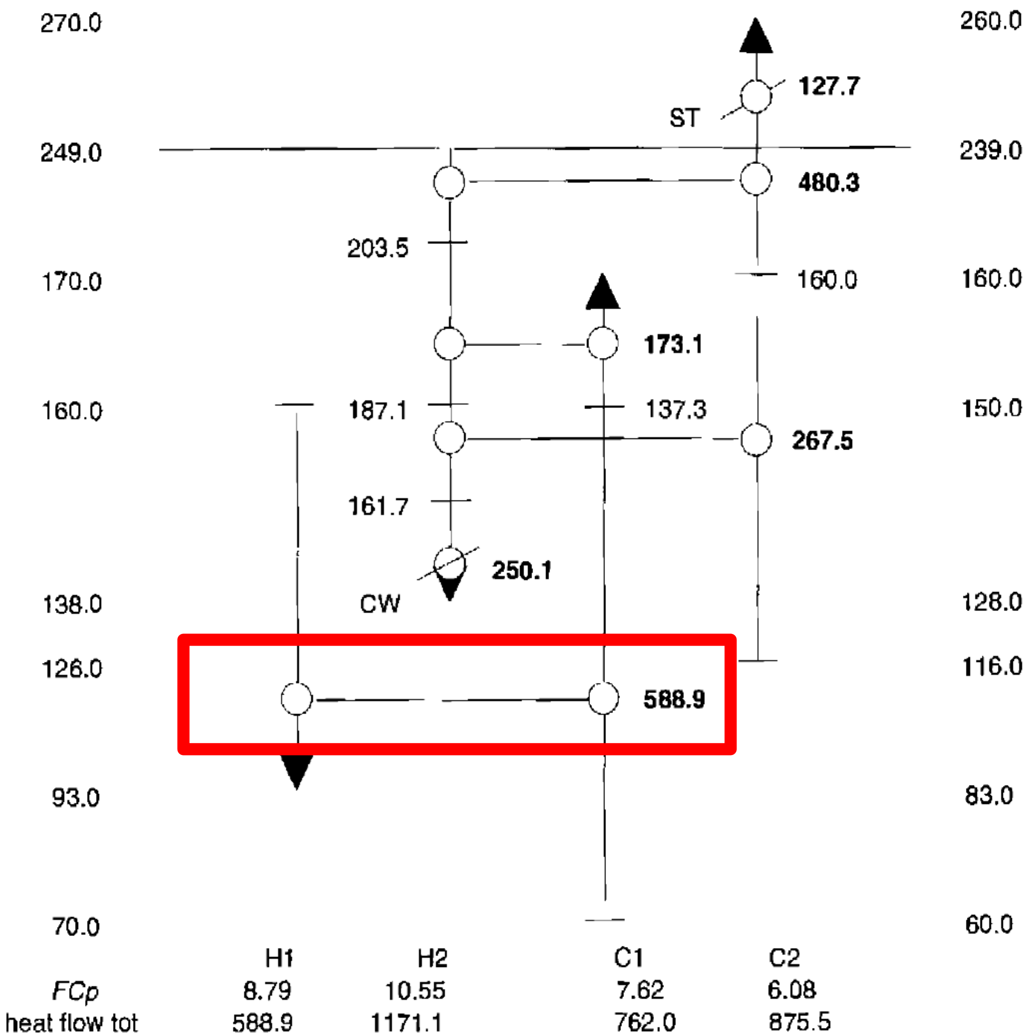
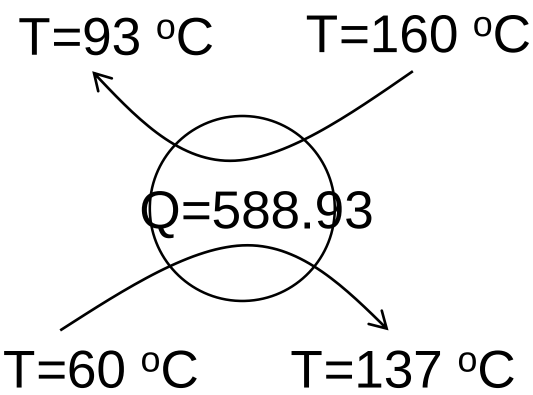


FIGURE 10.15 A possible heat exchanger network for 4SP1.



Use all H1 to heat C1

Heat Exchanger Network Synthesis (HENS) Problem

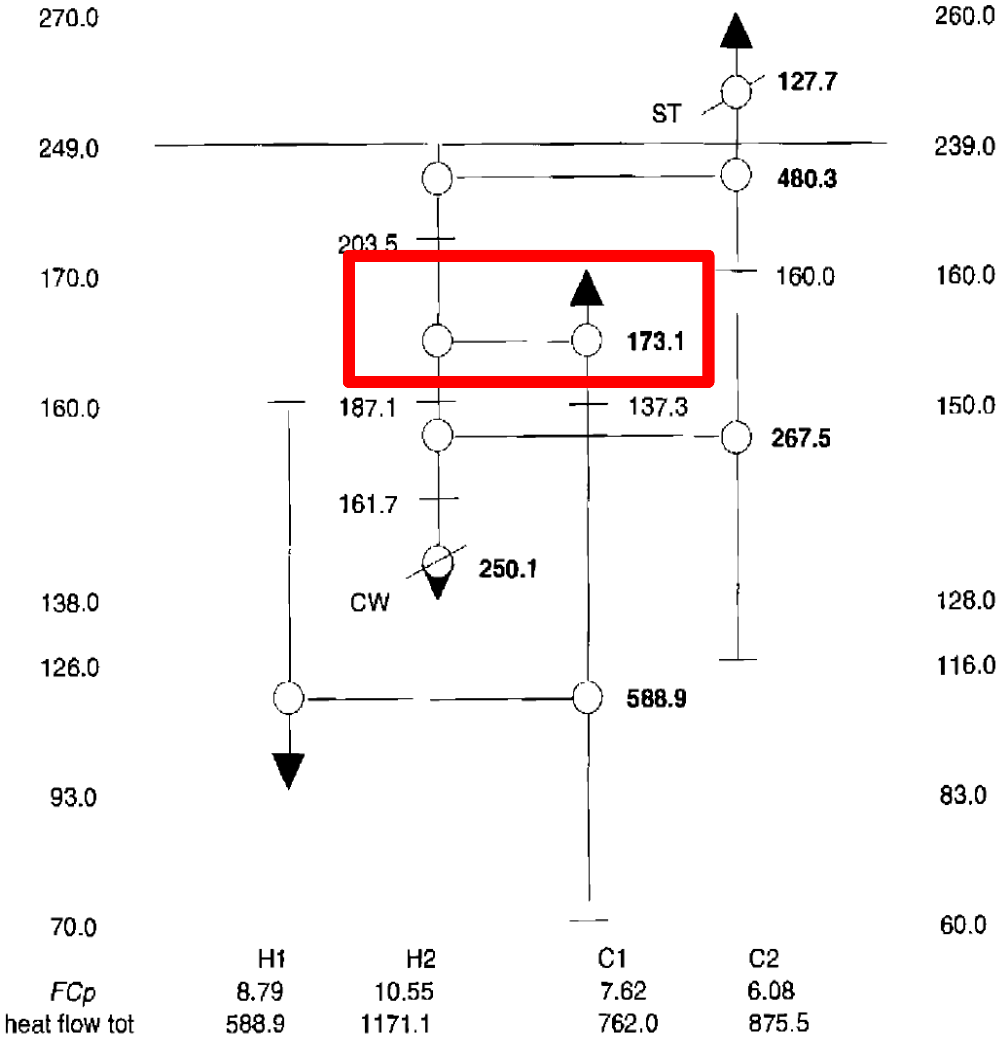
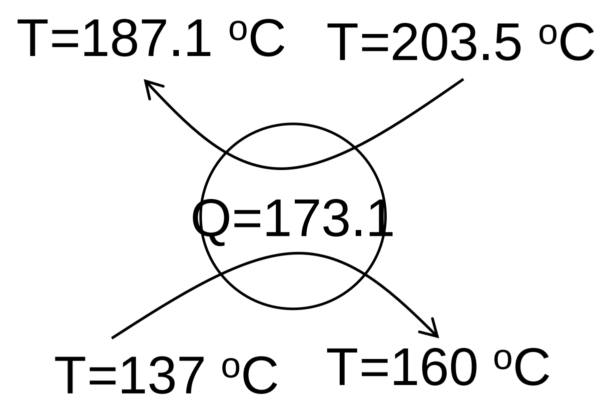


FIGURE 10.15 A possible heat exchanger network for 4SP1.

Heat C1 to target with H2



Heat Exchanger Network Synthesis (HENS) Problem

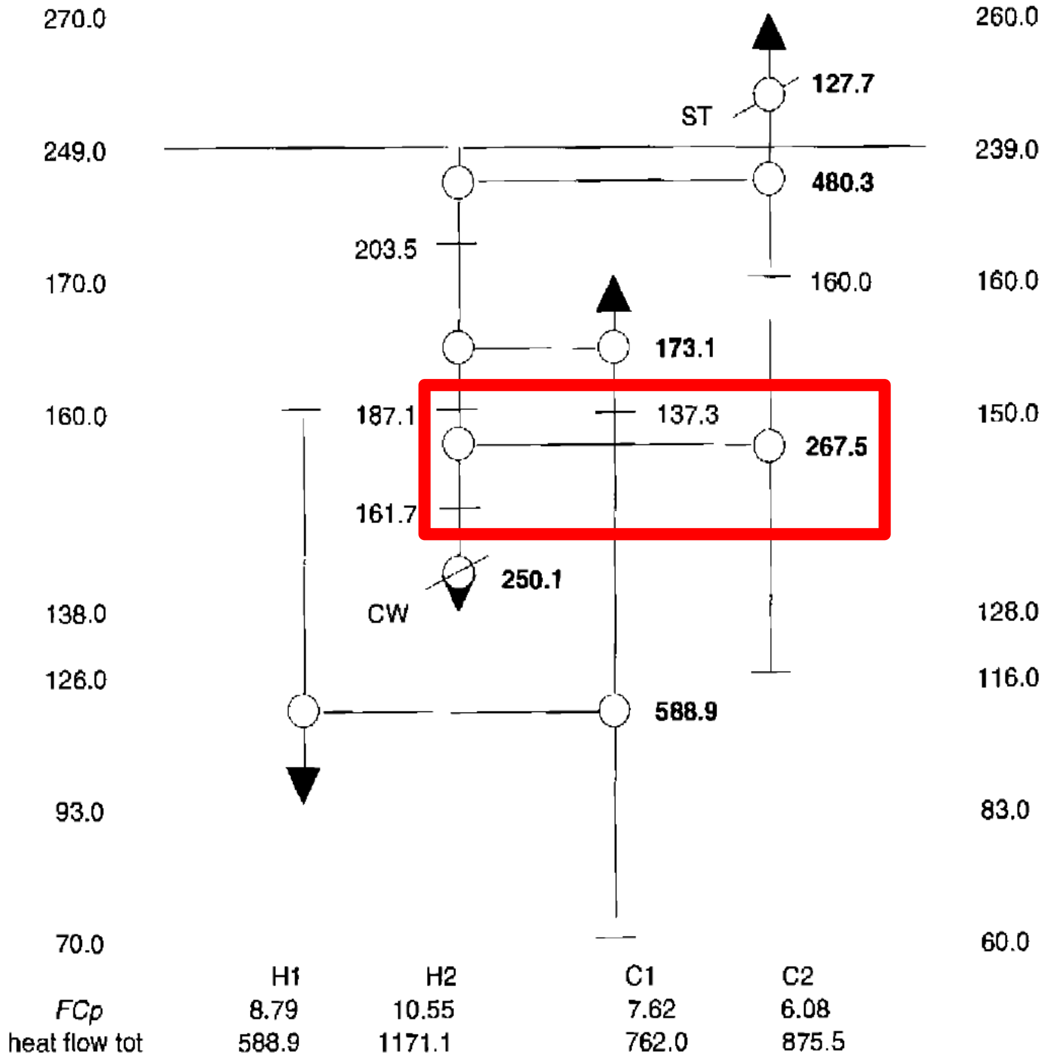
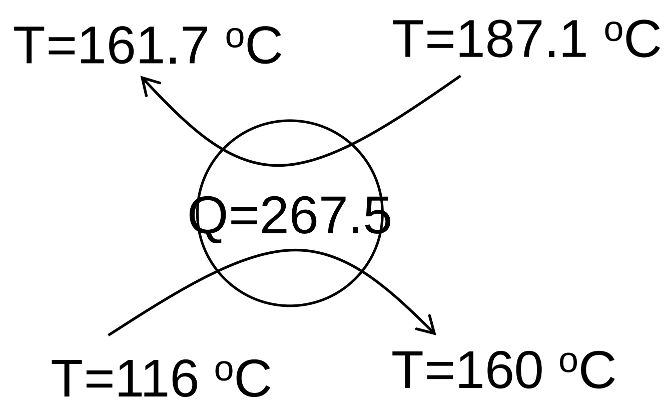


FIGURE 10.15 A possible heat exchanger network for 4SP1.

Start heating C2 with H2



Heat Exchanger Network Synthesis (HENS) Problem

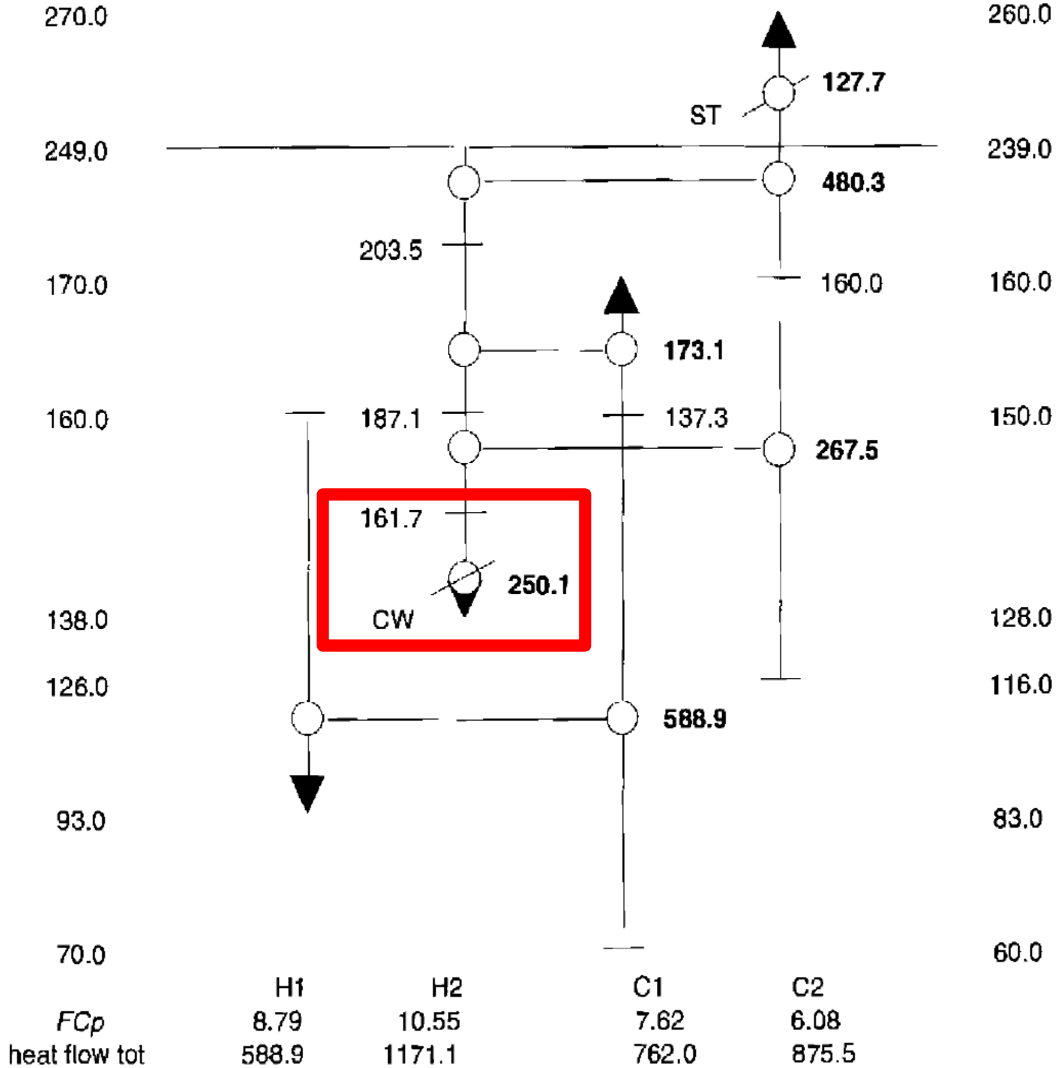
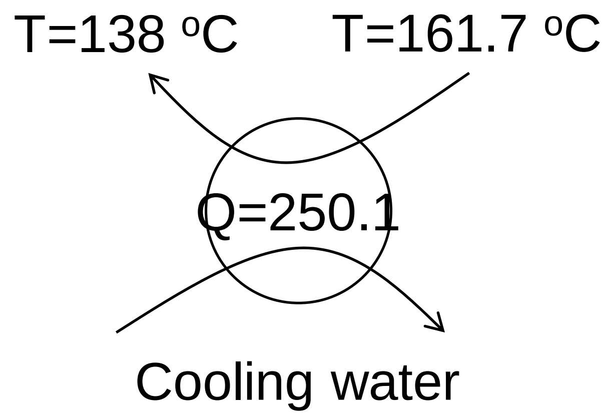
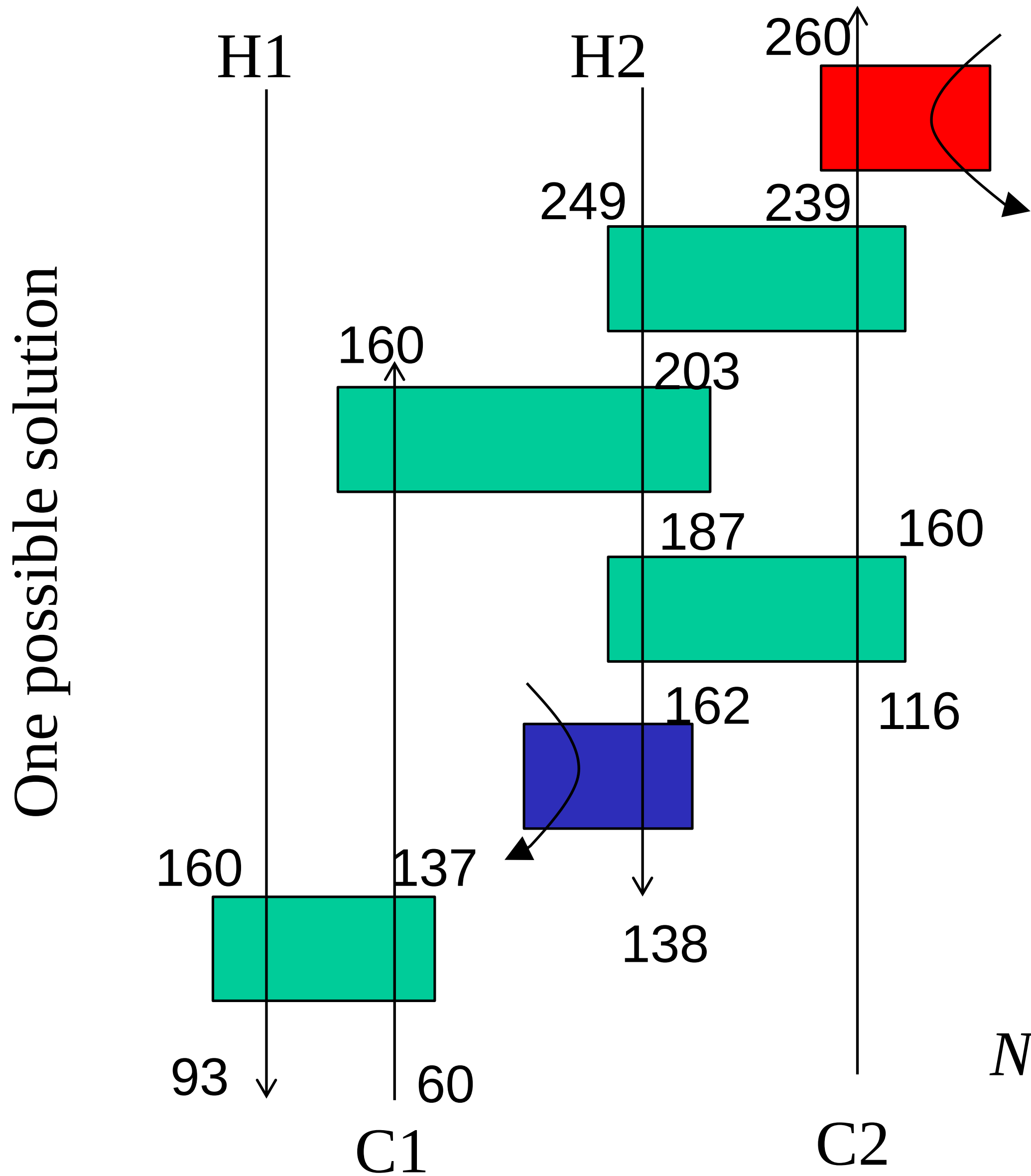


FIGURE 10.15 A possible heat exchanger network for 4SP1.



Cool H2 finally with cooling water

Heat Exchanger Network Synthesis (HENS) Problem



H2 and C2 are exchanged twice. Can we improve on this?

Split H2 and use one branch to heat all of C2 up to pinch while exactly reaching target for this branch

Needs to be evaluated further

Synthesis of heat exchanger networks: Synthesis strategies

Sequential optimization (LP, MILP)
Simultaneous optimization (MINLP) – own reading

Rules to be used in Sequential Optimization strategies

Rule 1: Minimum utility cost

Rule 2: Minimum number of units

Rule 3: Minimum investment costs

Minimum Utility Cost

Example 16.1

Determine the minimum utility consumption for the two hot and two cold streams given below:

	Fcp (MW/C)	Tin (C)	Tout (C)
H1	1	400	120
H2	2	340	120
C1	1.5	160	400
C2	1.3	100	250

Steam : 500°C
Cooling water: 20–30°C
Minimum recovery approach temperature (HRAT): 20°C

Solution steps

Step 1: **Create the interval table**

Step 2: **Add the heat contents to the interval table**

Step 3: **Convert the table from step 2 to a heat cascade diagram**

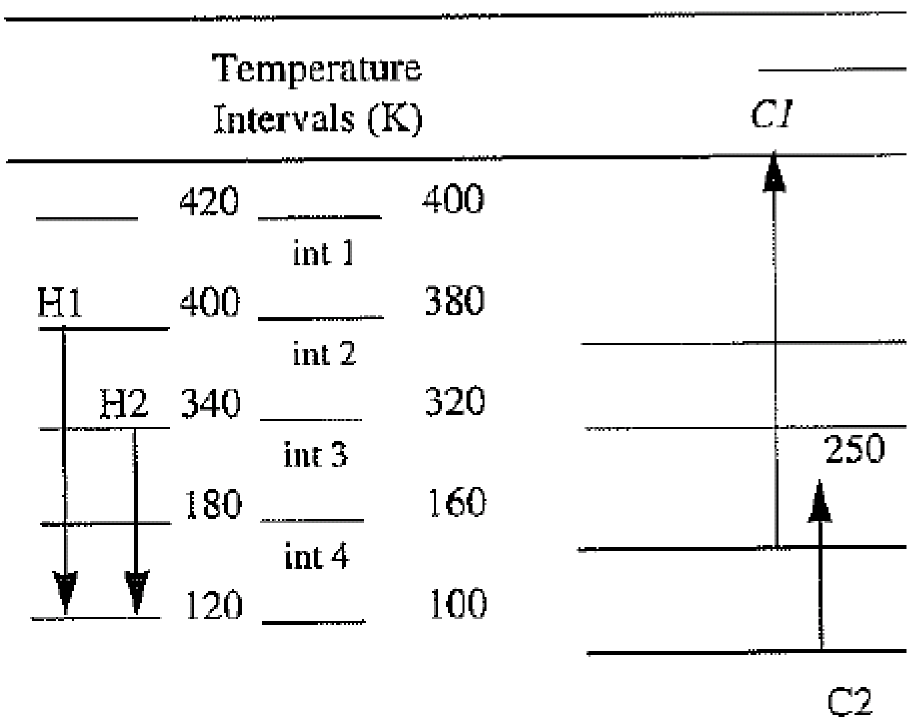
Step 4: **Derive energy balance equations for each interval on the heat cascade diagram**

Step 5: **Formulate and solve the LP optimization problem to find the pinch-point and the corresponding minimum utilities**

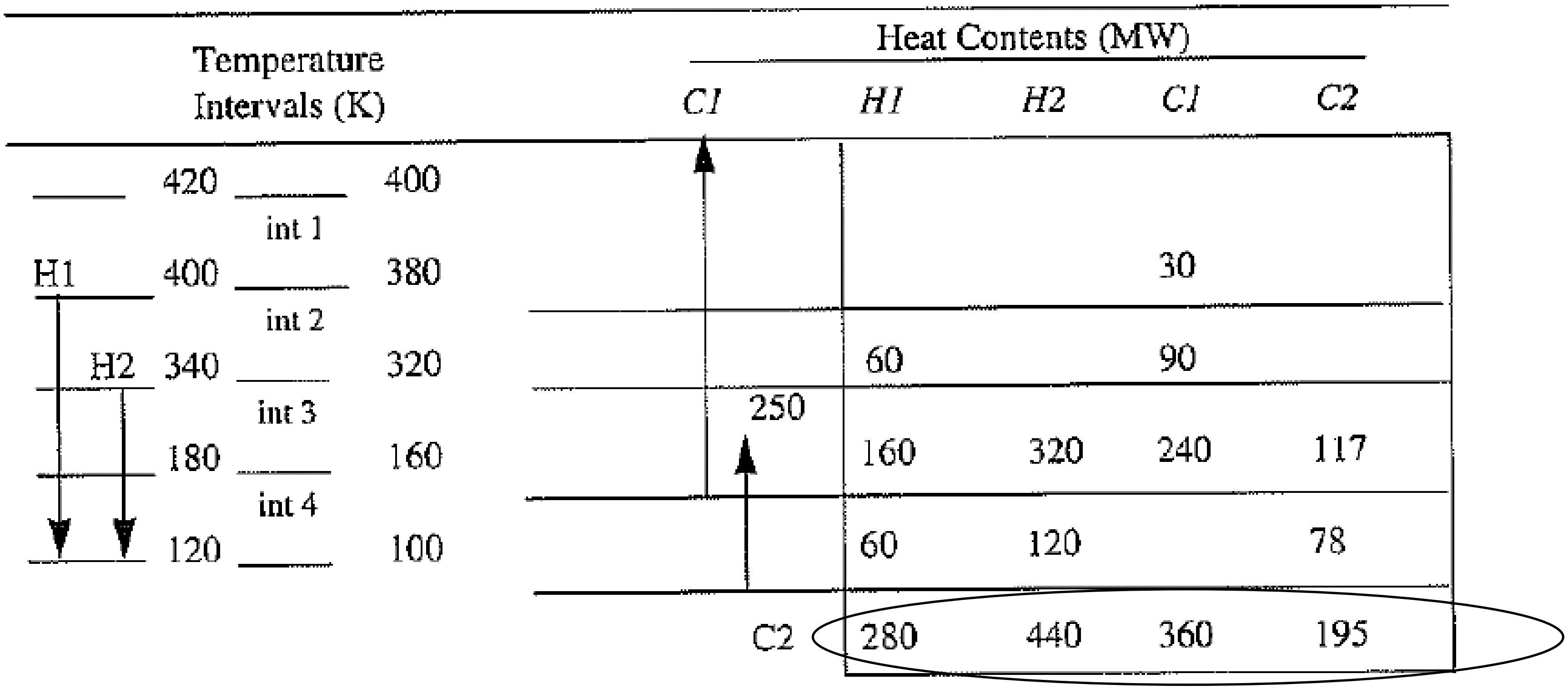
Solution steps – Step 1

	Fcp (MW/C)	Tin (C)	Tout (C)
H1	1	400	120
H2	2	340	120
C1	1.5	160	400
C2	1.3	100	250

ΔT = 20



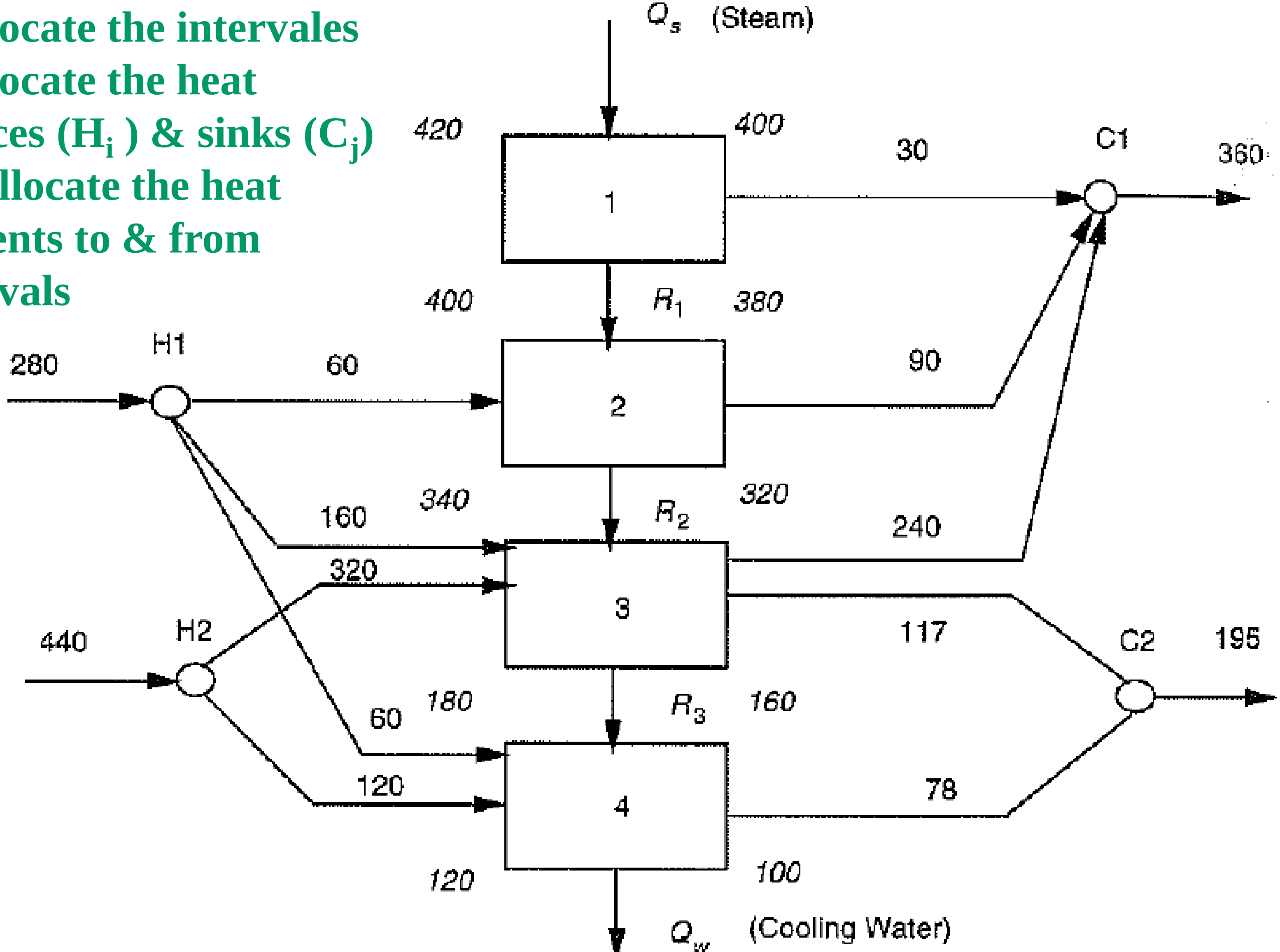
Solution steps – Step 2



Total heat to be removed or added

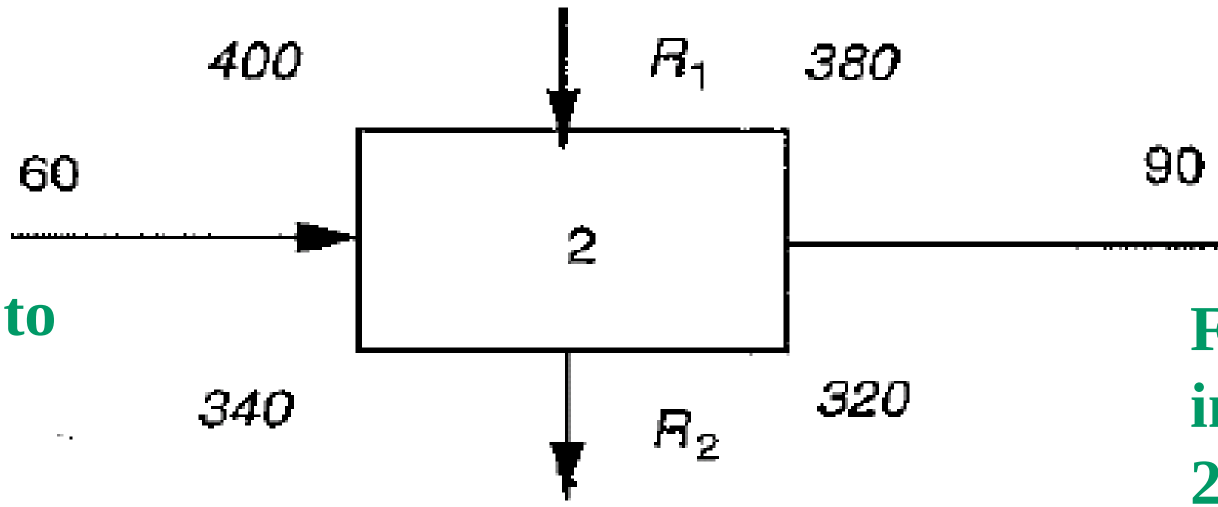
Solution steps – Step 3

1): Locate the intervales
2): Locate the heat sources (H_i) & sinks (C_j)
3): Allocate the heat contents to & from intervals



Solution steps – Step 4

For each interval, derive energy balance equation, for example, for interval 2



From H_1 to interval 2

From interval 2 to C_1

$$R_1 + 60 = R_2 + 90, \text{ that is,}$$
$$R_2 - R_1 = -30$$

Solution steps – Step 5

Collect all the equations for all intervals and add objective function (minimize utilities Q_S and Q_W) & solve the LP problem

$$\min Z = Q_s + Q_w$$

$$s.t. \quad R_1 - Q_s = -30$$

$$R_2 - R_1 = -30$$

$$R_3 - R_2 = 123$$

$$Q_w - R_3 = 102$$

$$Q_s, Q_w, R_1, R_2, R_3 \geq 0$$

$$\min Z = Q_s + Q_w$$

$$s.t. \quad R_1 - Q_s = -30$$

$$R_2 - R_1 = -30$$

$$R_3 - R_2 = 123$$

$$Q_w - R_3 = 102$$

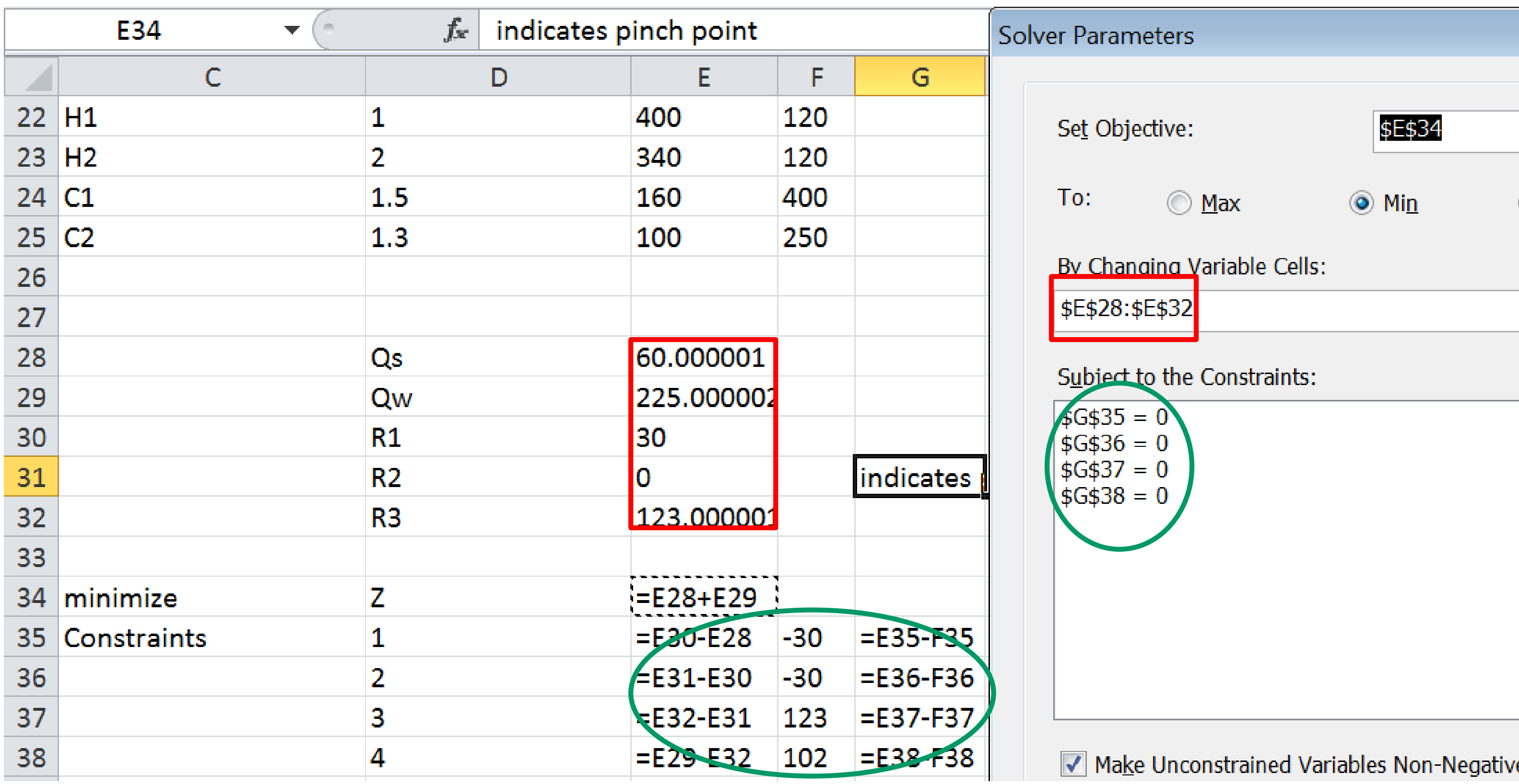
$$Q_s, Q_w, R_1, R_2, R_3 \geq 0$$

Solution steps – Step 5

Solution:

$R_1 = 30$; $R_2 = 0$; $R3 = 123$

$Q_S = 60$; $Q_W = 225$



E34 | indicates pinch point

	C	D	E	F	G
22	H1	1	400	120	
23	H2	2	340	120	
24	C1	1.5	160	400	
25	C2	1.3	100	250	
26					
27					
28		Qs	60.000001		
29		Qw	225.000002		
30		R1	30		
31		R2	0		indicates
32		R3	123.000001		
33					
34	minimize	Z	=E28+E29		
35	Constraints	1	=E30-E28	-30	=E35-F35
36		2	=E31-E30	-30	=E36-F36
37		3	=E32-E31	123	=E37-F37
38		4	=E29-E32	102	=E38-F38

Solution steps – Step 5

```
% we have 5 variable
% x1   x2   x3    x4    x5
% Qs   Qw   R1    R2    R3
n=5;
% objective function: x1+x2
f=zeros(n,1);
f(1)=1;
f(2)=1;
% inequality constraints: 0
A=[];
B=[];
% equality constraints: 4
neq=4;
Aeq=zeros(neq,n);
Beq=zeros(neq,1);
Aeq(1,1)=-1;    Aeq(1,3)=1;   Beq(1)=-30;   % -Qs + R1 = -30
Aeq(2,4)=1;     Aeq(2,3)=-1;  Beq(2)=-30;   %  R2 - R1 = -30
Aeq(3,5)=1;     Aeq(3,4)=-1;   Beq(3)=123;  %  R3 - R2 = 123
Aeq(4,2)=1;     Aeq(4,5)=-1;  Beq(4)=102;   %  Qw - R3 = 102

% boundary: non-negative
lb=zeros(n,1);

x=linprog(f,A,B,Aeq,Beq,lb)
```

Solution steps – Step 5

Collect all the equations for all intervals and add objective function (minimize utilities Q_S and Q_W) & solve the LP problem

$$\min Z = Q_s + Q_w$$

$$\begin{aligned} s.t.\quad & R_1 - Q_s = -30 \\ & R_2 - R_1 = -30 \\ & R_3 - R_2 = 123 \\ & Q_w - R_3 = 102 \\ & Q_s, Q_w, R_1, R_2, R_3 \geq 0 \end{aligned}$$

Rearrange → to obtain

$$\min Z = Q_s + Q_w$$

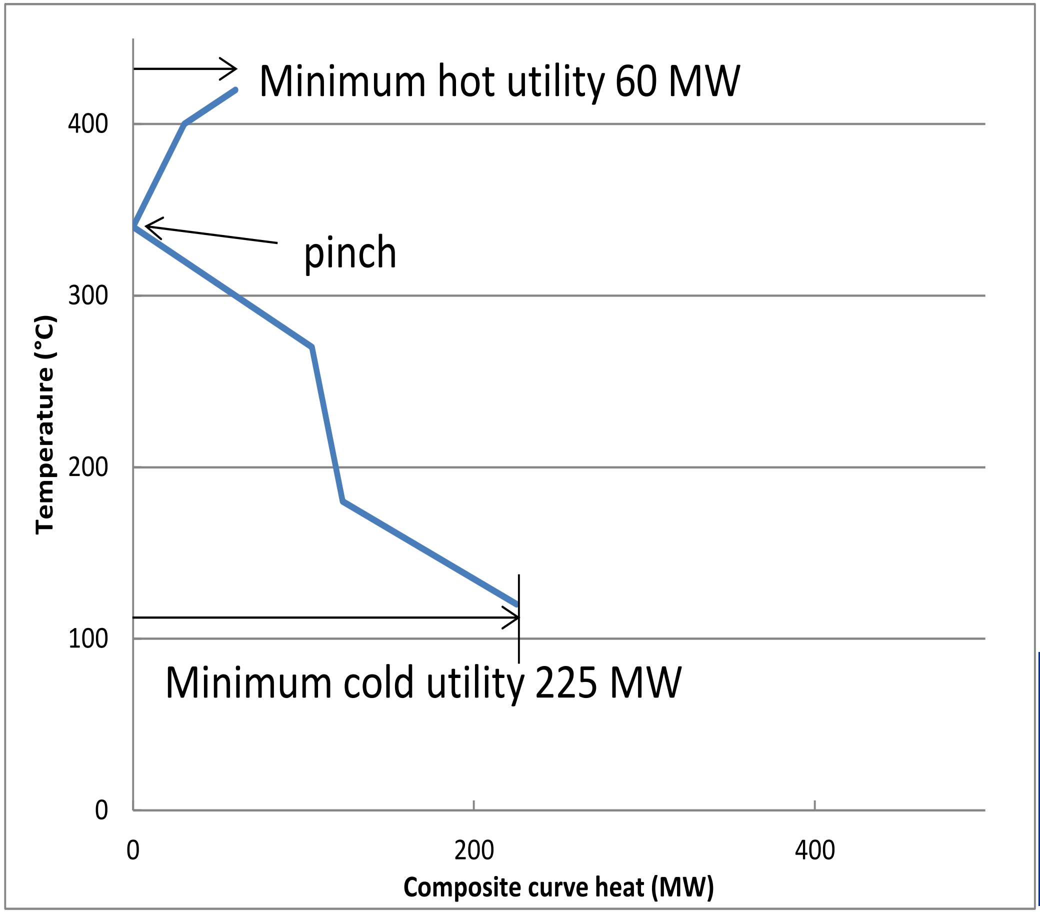
$$\begin{aligned} s.t.\quad & R_1 = Q_s - 30 \\ & R_2 = R_1 - 30 = Q_s - 60 \\ & R_3 = R_2 + 123 = Q_s + 63 \\ & Q_w = R_3 + 102 = Q_s + 165 \\ & R_1, R_2, R_3, Q_s, Q_w \geq 0 \end{aligned}$$

Solution:

$R_1 = 30$; $R_2 = 0$; R3 = 123

$Q_S = 60$; $Q_W = 225$

Minimum Utility



$$\min Z = Q_s + Q_w$$

$$s.t.\quad R_1 = Q_s - 30$$

$$R_2 = R_1 - 30 = Q_s - 60$$

$$R_3 = R_2 + 123 = Q_s + 63$$

$$Q_w = R_3 + 102 = Q_s + 165$$

$$R_1, R_2, R_3, Q_s, Q_w \geq 0$$

Solution:

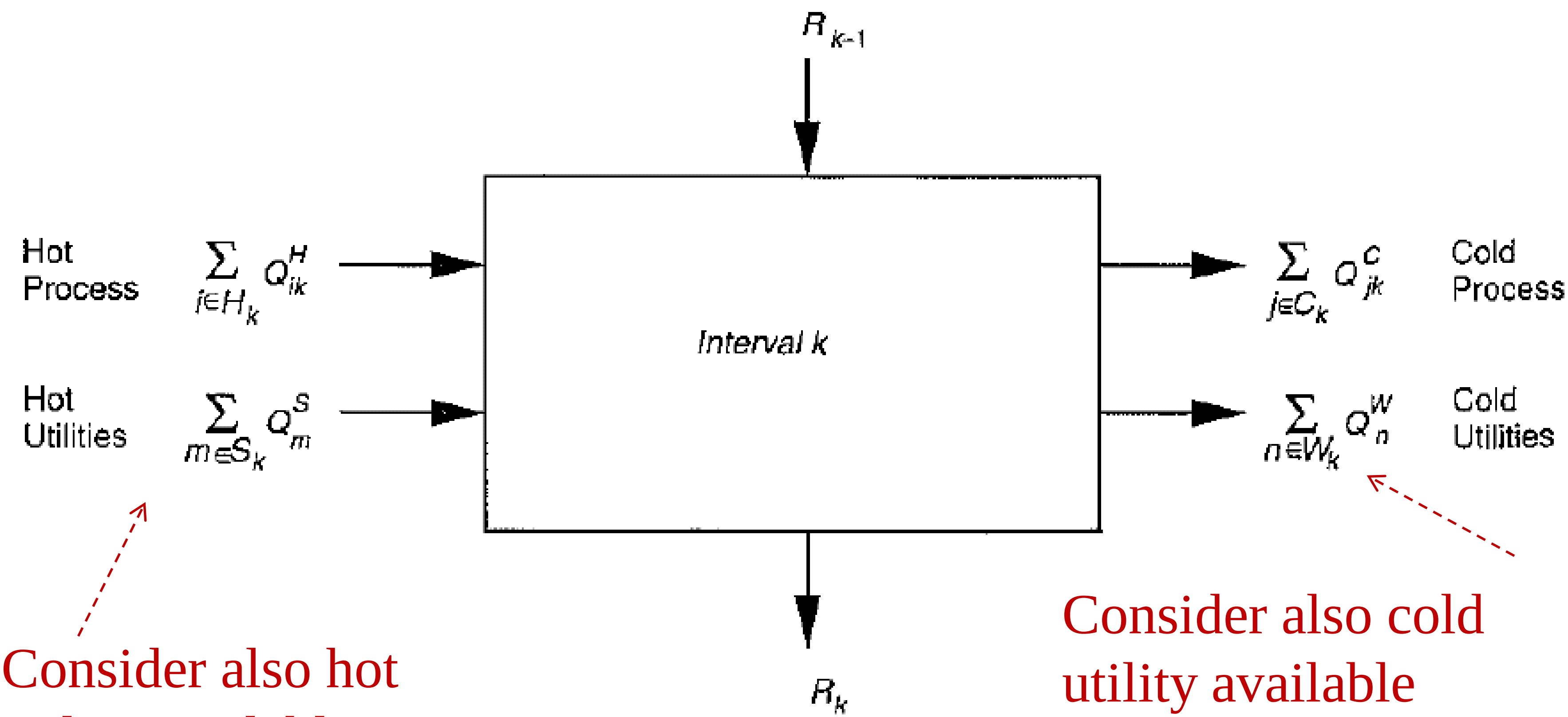
R_1 = 30; R_2 = 0; R3 = 123

Q_S =60; Q_W = 225

We have matched the results that can be obtained by the methods of chapter 10. However, we have not minimized the utility costs! The objective function needs a cost term.

Introduce the utility cost terms to form the LP transhipment model

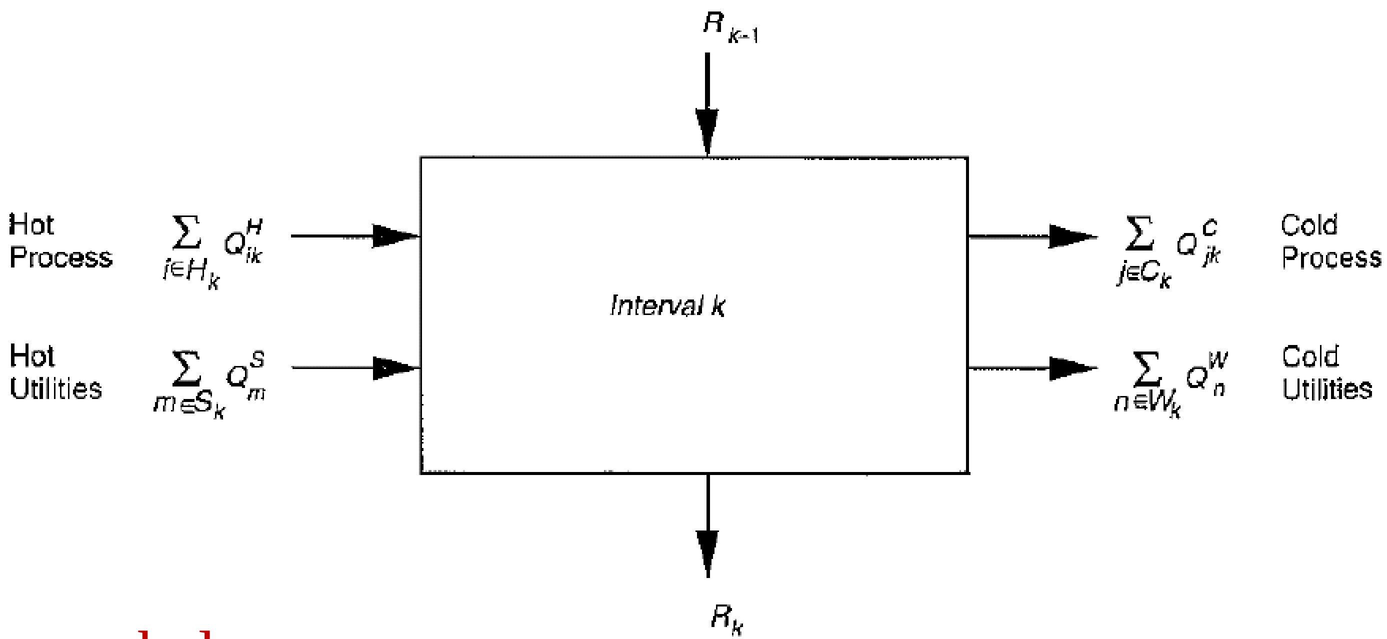
Add more details to each interval of the heat cascade diagram and derive new energy balance models



Consider also hot utility available

Consider also cold utility available

Introduce the utility cost terms to form the LP transhipment model



New energy balance for interval k

$$\min \; Z = \sum_{m \in S} c_m Q_m^S + \sum_{n \in W} c_n Q_n^W$$

$$s.t. \; R_k - R_{k-1} - \sum_{m \in S_k} Q_m^S + \sum_{n \in W_k} Q_n^W = \sum_{i \in H_k} Q_{ik}^H - \sum_{j \in C_k} Q_{jk}^C \qquad k = 1, \dots K$$

$$Q_m^S \geq 0 \quad Q_n^W \geq 0 \quad R_k \geq 0 \quad k = 1, \dots K-1$$

$$R_0 = 0, R_K = 0$$

Minimum utility cost with new problem formulation

Example 16.2: Given the data in Table 16.2 for two hot-streams and two cold-streams, two hot utilities and one cold utility, Determine the minimum utility cost with the LP transhipment problem

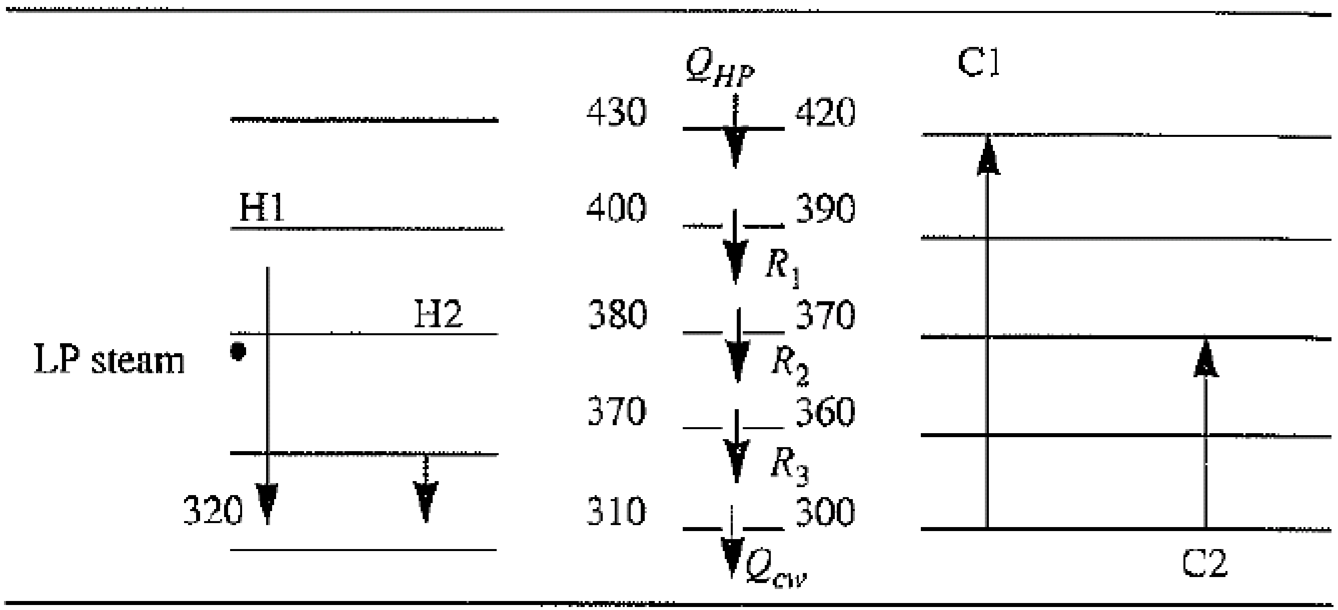
TABLE 16.2 Data for Example 16.2

	FCp (MW/K)	T_{in}(K)	T_{out}(K)
H1	2.5	400	320
H2	3.8	370	320
C1	2	300	420
C2	2	300	370

HP Steam: 500K $80/kWyr
LP Steam: 380K $50/kWyr
Cooling Water: 300K $20/kWyr
Minimum Recovery Approach Temperature (HRAT): 10K

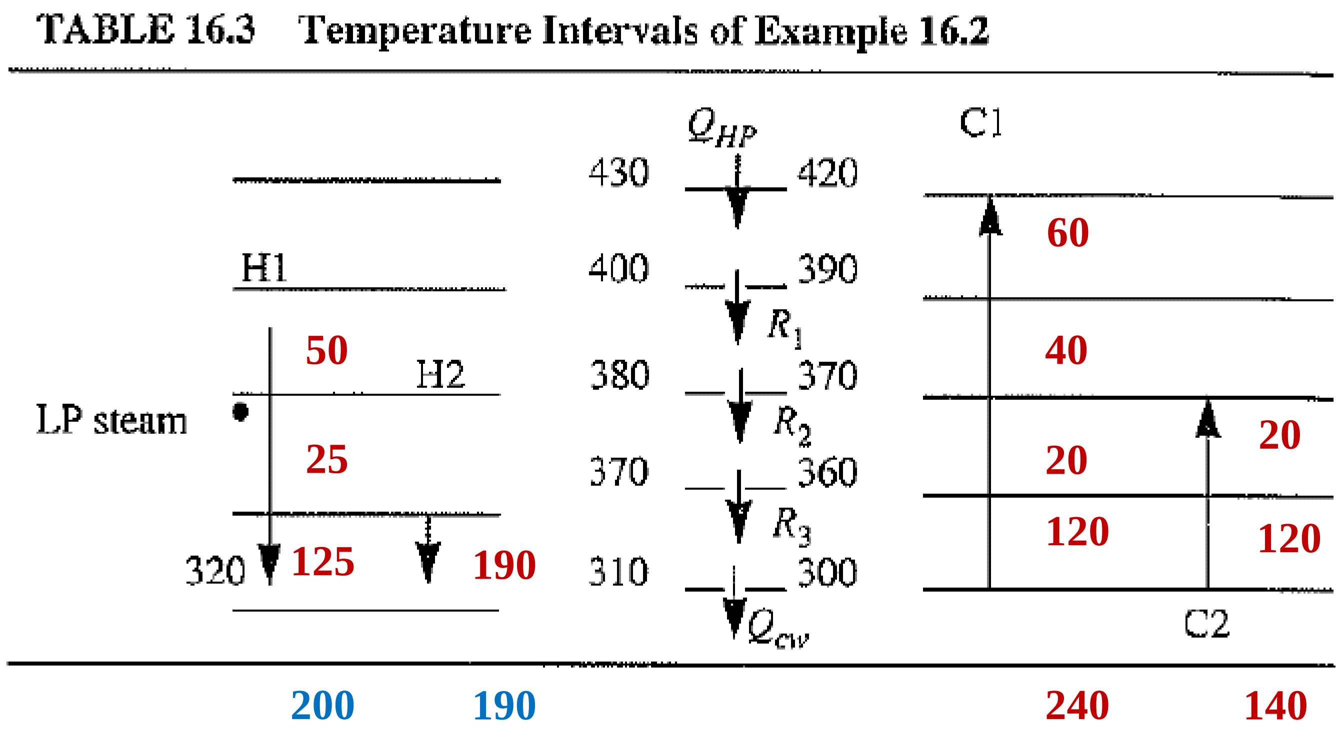
Minimum Utility Cost: LP Transhipment model (example 16.2) : Step 1

TABLE 16.3 Temperature Intervals of Example 16.2



Perform steps 2-4 (exercise in class)

Minimum Utility Cost: LP Transhipment model (example 16.2) : Step 1



Perform steps 2-4 (exercise in class)

Minimum Utility Cost: LP Transhipment model (example 16.2) : Step 5

$$\min Z = 80000\, Q_{HP} + 50000\, Q_{LP} + 20000\, Q_{CW}$$

$$\begin{aligned} s.t. \quad & R_1 - Q_{HP} = -60 \\ & R_2 - R_1 = 10 \\ & R_3 - R_2 - Q_{LP} = -15 \\ & -R_3 + Q_{CW} = 75 \\ & R_1, R_2, R_3, Q_{HP}, Q_{LP}, Q_{CW} \geq 0 \end{aligned}$$

→

The solution to this LP yields the following results:

Utility cost: $Z = 6{,}550{,}000$ \$/yr.

Heat load high pressure steam: $Q_{HP} = 60\ MW$

Heat load low pressure steam: $Q_{LP} = 5\ MW$

Heat load cooling water: $Q_{CW} = 75\ MW$

Residuals: $R_1 = 0$, $R_2 = 10\ MW$, $R_3 = 0$.

↙

The two above zero residuals imply that there are two pinch points for this problem: at 400–390 K, and at 370–360 K. This means that the temperature intervals in this problem can be partitioned into three subnetworks:

Subnetwork 1:	above 400–390 K
Subnetwork 2:	between 400–390 K and 370–360 K
Subnetwork 3:	below 370–360 K

Minimum Utility Cost: LP Transhipment model (example 16.2) : Step 5

$$\min Z = 80000\, Q_{HP} + 50000\, Q_{LP} + 20000\, Q_{CW}$$

$$\begin{aligned} s.t.\quad & R_1 - Q_{HP} = -60 \\ & R_2 - R_1 = 10 \\ & R_3 - R_2 - Q_{LP} = -15 \\ & -R_3 + Q_{CW} = 75 \\ & R_1, R_2, R_3, Q_{HP}, Q_{LP}, Q_{CW} \geq 0 \end{aligned}$$

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Subnetwork 1:	above 400–390 K
Subnetwork 2:	between 400–390 K and 370–360 K
Subnetwork 3:	below 370–360 K

Assumes any given pair of hot & cold streams can exchange heat! This may not be true

Minimum Utility Cost with Constrained Matches

The LP transhipment model implicitly assumes that any given pair of hot and cold streams can exchange heat (because no information as to which pair can or cannot exchange heat was included)

1. Transportation model where we consider directly all the feasible links for heat exchange between each pair of hot and cold streams over their corresponding temperature intervals (Cerda and Westerberg, 1983). Figure 16.3 illustrates this representation for Example 16.1.

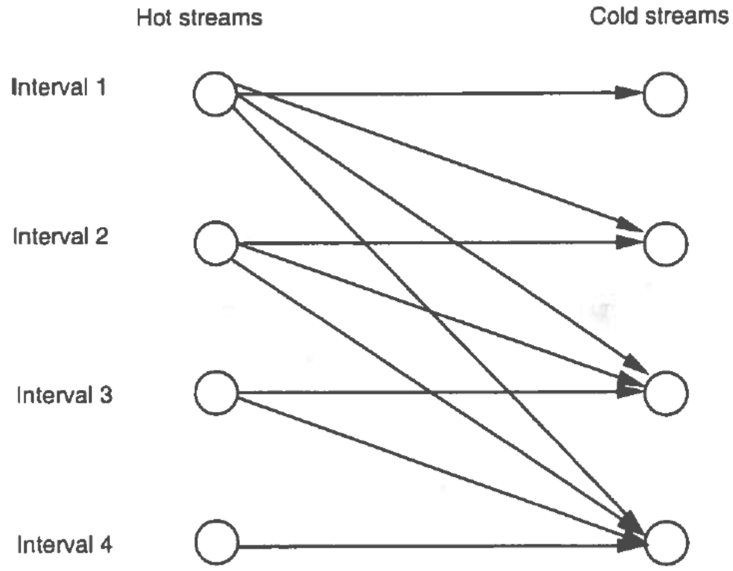


FIGURE 16.3 Representation of heat flows for transportation model.

Minimum Utility Cost with Constrained Matches

The LP transhipment model implicitly assumes that any given pair of hot and cold streams can exchange heat (because no information as to which pair can or cannot exchange heat was included)

1. Transportation model where we consider directly all the feasible links for heat exchange between each pair of hot and cold streams over their corresponding temperature intervals (Cerda and Westerberg, 1983). Figure 16.3 illustrates this representation for Example 16.1.

2. Expanded transshipment model (Papoulias and Grossmann, 1983) where we consider within each temperature interval a link for the heat exchange between a given pair of hot and cold streams, where the cold stream is present at that interval and the hot stream is either also present, or else it is present in a higher temperature interval. Figure 16.4 illustrates this representation for Example 16.1.

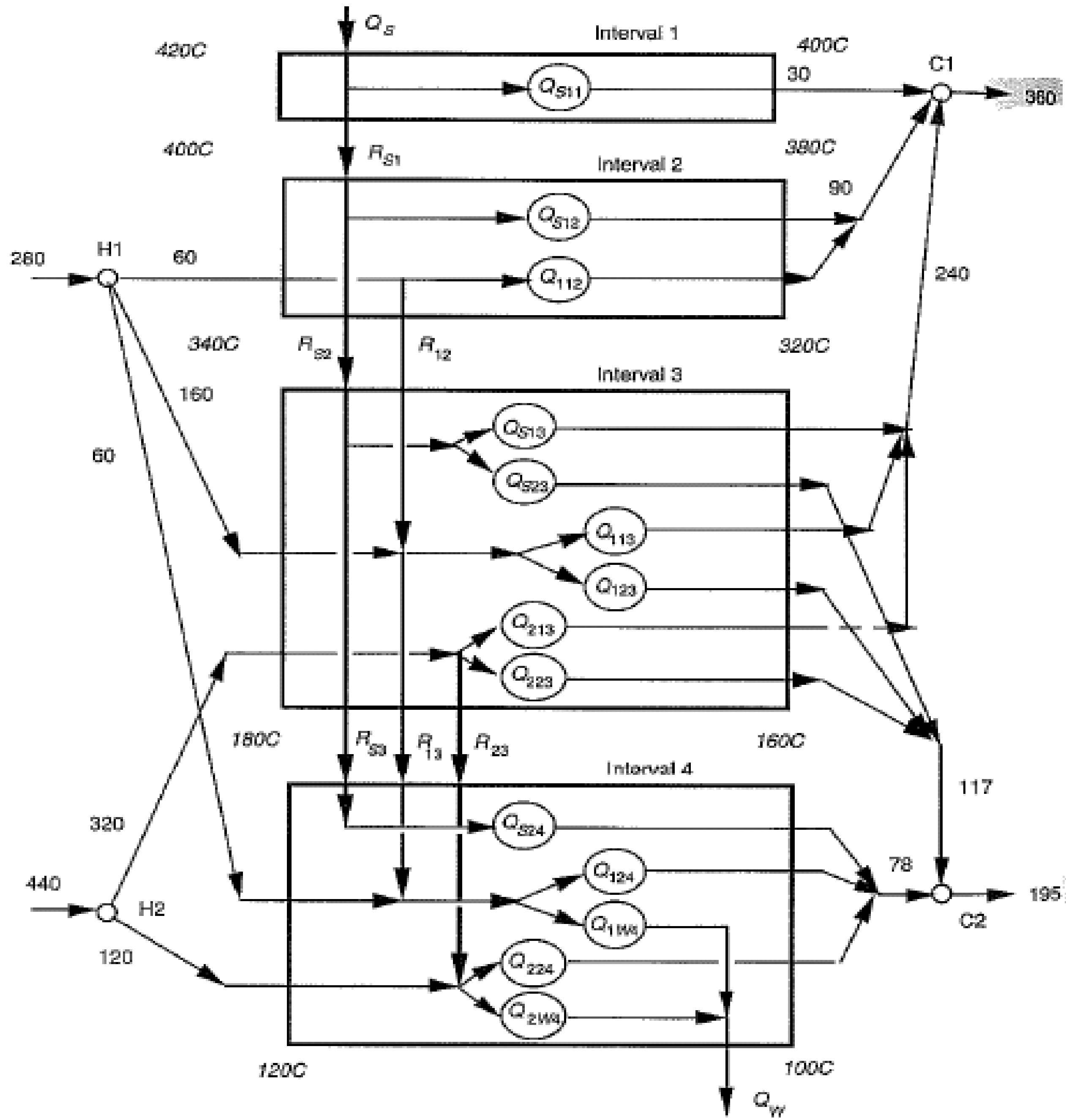
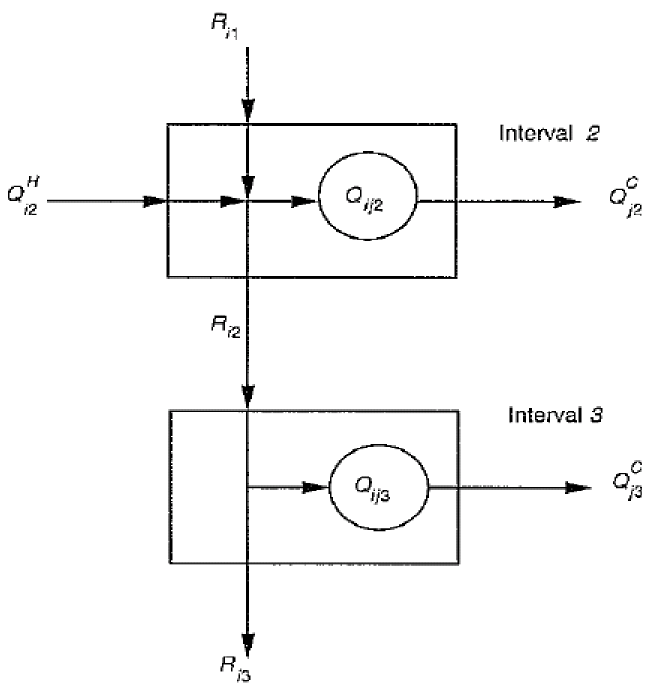


FIGURE 16.4 Representation of expanded transshipment model for Example 16.1.

Minimum Utility Cost with Constrained Matches: Extension of the LP transhipment model

The basic idea in the expanded transshipment model is as follows. First, instead of assigning a single overall heat residual R_k exiting at each temperature level k, we will assign individual heat residuals R_{ik}, R_{mk} for each hot stream i and each hot utility m that are present at or above that temperature interval k. Secondly, within that interval k we will define the variable Q_{ijk} to denote the heat exchange between hot stream i and a cold stream j. Likewise, we can define similar variables for the exchange between process streams and utilities



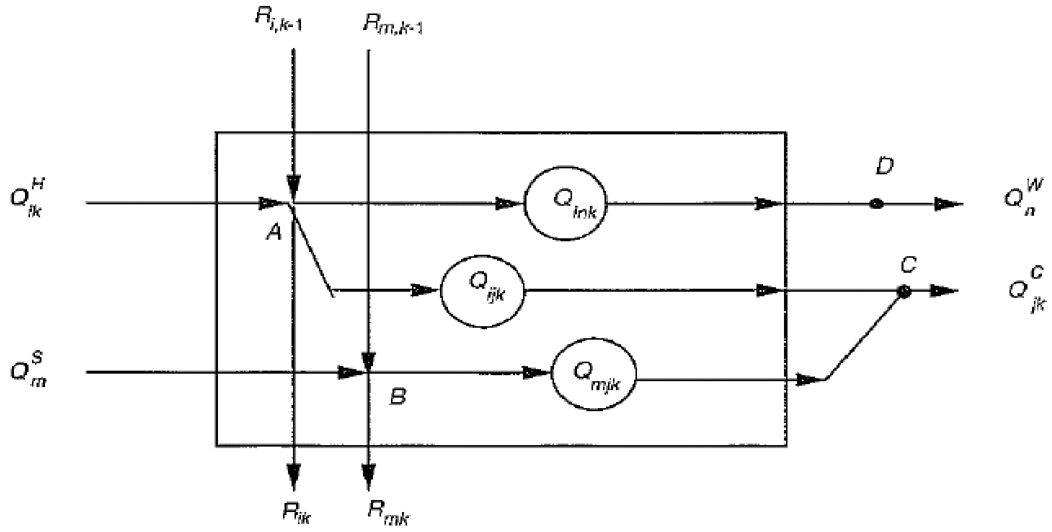
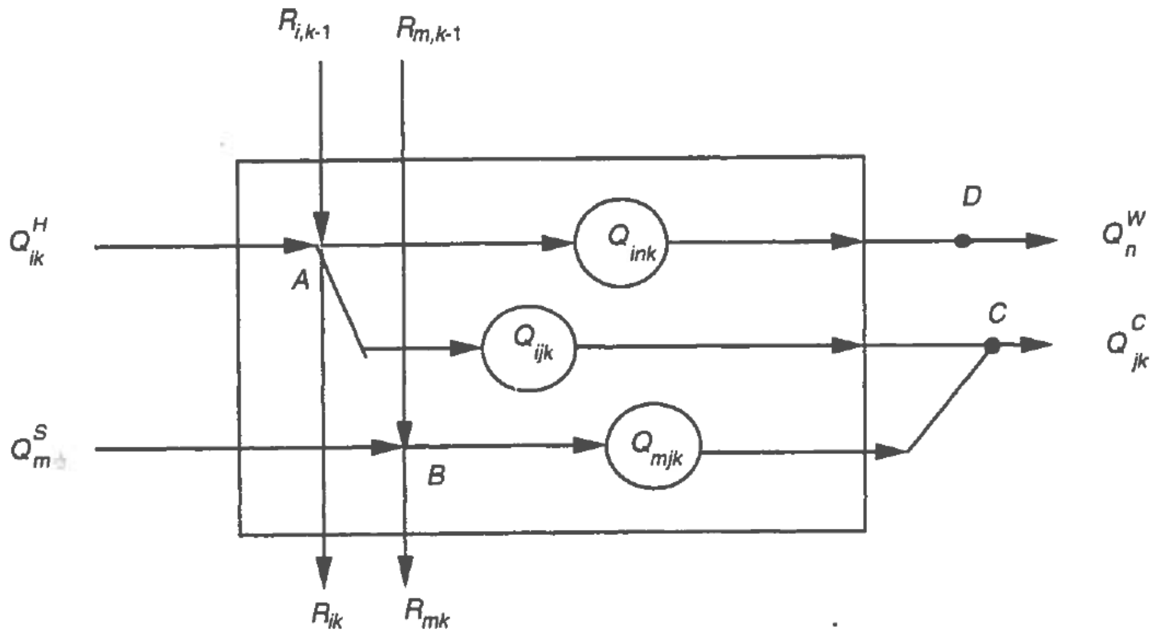


FIGURE 16.7 Heat flows in expanded transshipment model.

The new more detailed model

k: interval
i: hot stream
j: cold stream
m: hot utility
n: cold utility



$$\min \; Z = \sum_{m \in S} c_m Q^S_m + \sum_{n \in W} c_n Q^W_n$$

$$\text{s.t.} \quad R_{ik} - R_{i,k-1} + \sum_{j \in C_k} Q_{ijk} + \sum_{n \in W_k} Q_{ink} = Q^H_{ik} \qquad i \in H'_k$$

$$R_{mk} - R_{m,k-1} + \sum_{j \in C_k} Q_{mjk} - Q^S_m = 0 \quad m \in S'_k$$

$$\sum_{i \in H_k} Q_{ijk} + \sum_{m \in S_k} Q_{mjk} = Q^C_{jk} \quad j \in C_k$$

$$\sum_{i \in H_k} Q_{ink} - Q^W_n = 0 \qquad n \in W_k \quad k = 1, \ldots K$$

$$R_{ik},\ R_{mk},\ Q_{ijk},\ Q_{mjk},\ Q_{ink},\ Q^S_m,\ Q^W_n \geq 0$$

$$R_{i0} = R_{iK} = 0$$

Advantages

$$\min \ z = \sum_{m \in S} c_m Q_m^S + \sum_{n \in W} c_n Q_n^W$$

$$\text{s.t.} \quad R_{ik} - R_{i,k-1} + \sum_{j \in C_k} Q_{ijk} + \sum_{n \in W_k} Q_{ink} = Q_{ik}^H \qquad i \in H_k'$$

$$R_{mk} - R_{m,k-1} + \sum_{j \in C_k} Q_{mjk} - Q_m^S = 0 \quad m \in S_k'$$

$$\sum_{i \in H_k} Q_{ijk} + \sum_{m \in S_k} Q_{mjk} = Q_{jk}^C \quad j \in C_k$$

$$\sum_{i \in H_k} Q_{ink} - Q_n^W = 0 \qquad n \in W_k \quad k = 1, \dots K$$

$$R_{ik}, \ R_{mk}, \ Q_{ijk}, \ Q_{mjk}, \ Q_{ink}, \ Q_m^S, \ Q_n^W \geq 0$$

$$R_{i0} = R_{iK} = 0$$

The size of this LP is obviously larger than the previous one. We can very easily specify constratins on the matches!

If we want to forbid a match between hot stream *i* and cold stream *j*, all we need to do is to set $Q_{ijk}=0$ for all intervals k.

Minimum Utility Cost & Constrained Matching

EXAMPLE 16.3

Let us consider the example in Table 16.1 that we examined in section 16.2. For that example we found that by not imposing any restriction on the matches, the minimum heating is 60 MW, and the minimum cooling is 225 MW. If the cost of the heating and cooling utilities is \$80/kWyr and \$20/kWyr, respectively, this would mean an annual cost of \$9,300,000/yr. In addition, we found a pinch point at 340–320°C. Let us assume now that we were to impose as a constraint that the match for stream H1 and C1 is forbidden.

Temperature Intervals (K)				Heat Contents (MW)				
Streams	Hot	Interval	Cold	C1	H1	H2	C1	C2
	420		400					
		int 1					30	
H1	400		380					
		int 2			60		90	
H2	340		320					
		int 3		250	160	320	240	117
	180		160					
		int 4			60	120		78
	120		100					
				C2	280	440	360	195

- Convert to the new detailed heat cascade diagram (Fig 16.4) –

Compare this diagram with Fig 16.1

- Derive the energy balance equations for each interval with the extended transhipment model

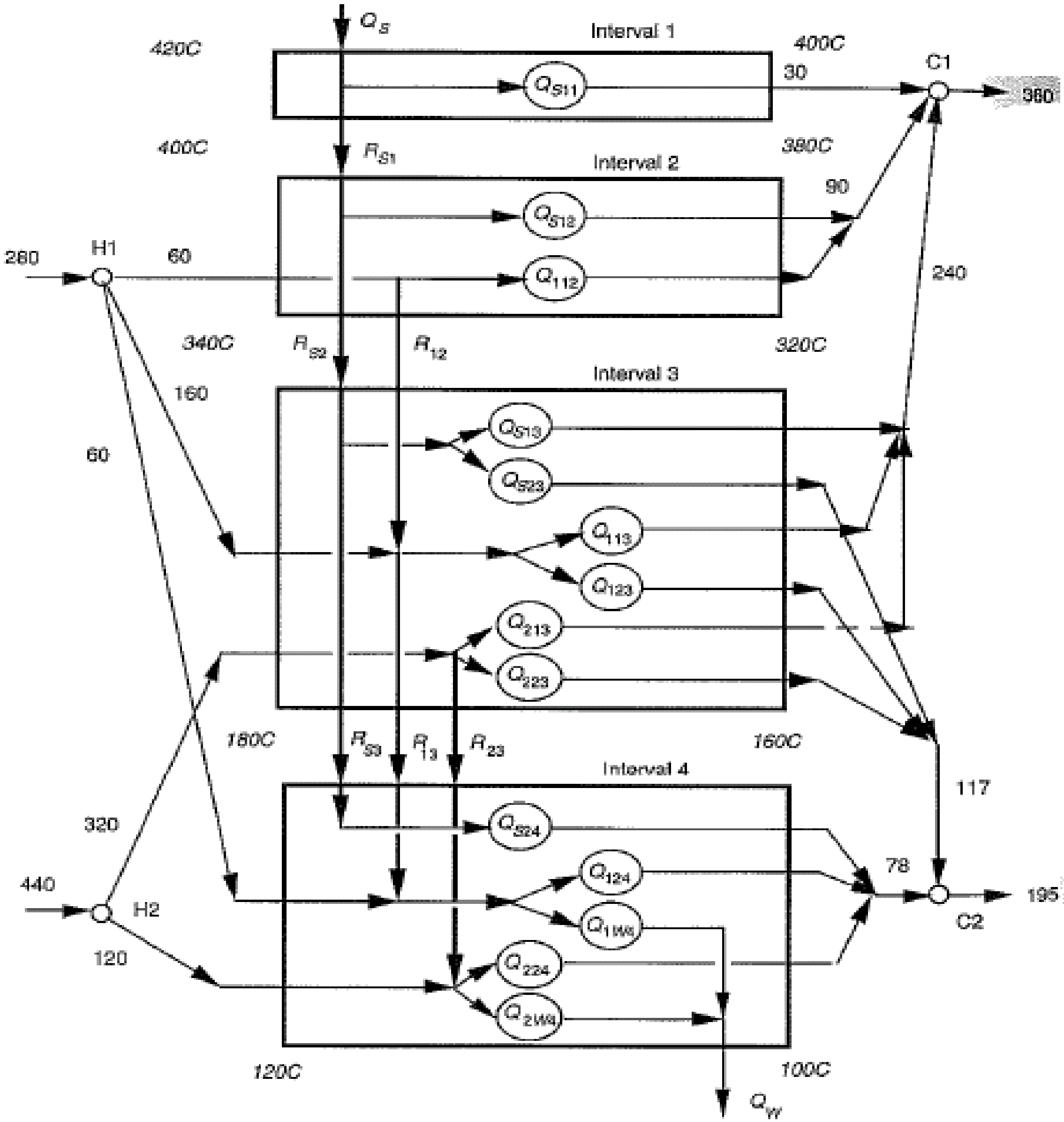


FIGURE 16.4 Representation of expanded transshipment model for Example 16.1.

Exercise in class

- **Convert to the new detailed heat cascade diagram (Fig 16.4) –**

Compare this diagram with Fig 16.1

- **Derive the energy balance equations for each interval with the extended transhipment model**

Exercise in class

TABLE 16.4 Expanded LP for Restricted Match in Example 16.3

Utility Cost:		$\min Z = 80000\, Q_S + 20000 Q_W$
Interval 1:	s.t.	$R_{S1} + Q_{S11} - Q_S = 0$
		$Q_{S11} = 30$
Interval 2:		$R_{12} + Q_{112} = 60$
		$R_{S2} - R_{S1} + Q_{S12} = 0$
		$Q_{S12} + Q_{112} = 90$
Interval 3:		$R_{13} - R_{12} + Q_{113} + Q_{123} = 160$
		$R_{23} + Q_{213} + Q_{223} = 320$
		$R_{S3} - R_{S2} + Q_{S13} + Q_{S23} = 0$
		$Q_{113} + Q_{213} + Q_{S13} = 240$
		$Q_{123} + Q_{223} + Q_{S23} = 117$
Interval 4:		$-R_{13} + Q_{124} + Q_{1W4} = 60$
		$-R_{23} + Q_{224} + Q_{2W4} = 120$
		$-R_{S3} + Q_{S24} = 0$
		$Q_{124} + Q_{224} + Q_{S24} = 78$
		$Q_{1W4} + Q_{2W4} - Q_W = 0$
Forbidden match:		$Q_{112} = Q_{113} = 0$ (H1–C1 do not exchange heat)

Solving it in EXCEL

cost steam	80000				
cost water	20000				
cost utilities		=C42*E28+C43*E29	$/y		=C42*D50+C43*D51
Transshipment model					
			Constraints		
	Qs	120	=D52+D58-D50	0	=E50-F50
	Qw	285.000001	=D58	30	=E51-F51
	Rs1	89.9999999999999	=D55	60	=E52-F52
	Rs2	0	=D53-D52+D59	0	=E53-F53
	Rs3	0	=D59	90	=E54-F54
	R12	60	=D56-D55+D62	160	=E55-F55
	R13	103.000001	=D57+D63+D64	320	=E56-F56
	R23	80	=D54-D53+D60+D61	0	=E57-F57
	Qs11	30	=D63+D60	240	=E58-F58
	Qs12	90	=D62+D64+D61	117	=E59-F59
	Qs13	0	=D66+D67-D56	60	=E60-F60
	Qs23	0	=D68+D69-D57	120	=E61-F61
	Q123	116.999999	=D65-D54	0	=E62-F62
	Q213	240	=D66+D68+D65	78	=E63-F63
	Q223	0	=D67+D69-D51	0	=E64-F64
	Qs24	0	=D70	=0	=E65-F65
	Q124	78	=D71	=0	=E66-F66
	Q1w4	85.0000010000001			
	Q224	0			
	Q2w4	200			
	Q112	0			
	Q113	0			

TABLE 16.4 Expanded LP for Restricted Match in Example 16.3

Utility Cost: $\min Z = 80000\, Q_S + 20000 Q_W$

Interval 1: s.t. $R_{S1} + Q_{S11} - Q_S = 0$

$Q_{S11} = 30$

Interval 2: $R_{12} + Q_{112} = 60$

$R_{S2} - R_{S1} + Q_{S12} = 0$

$Q_{S12} + Q_{112} = 90$

Interval 3: $R_{13} - R_{12} + Q_{113} + Q_{123} = 160$

$R_{23} + Q_{213} + Q_{223} = 320$

$R_{S3} - R_{S2} + Q_{S13} + Q_{S23} = 0$

$Q_{113} + Q_{213} + Q_{S13} = 240$

$Q_{123} + Q_{223} + Q_{S23} = 117$

Interval 4: $-R_{13} + Q_{124} + Q_{1W4} = 60$

$-R_{23} + Q_{224} + Q_{2W4} = 120$

$-R_{S3} + Q_{S24} = 0$

$Q_{124} + Q_{224} + Q_{S24} = 78$

$Q_{1W4} + Q_{2W4} - Q_W = 0$

Forbidden match: $Q_{112} = Q_{113} = 0$ (H1–C1 do not exchange heat)

$15,300,000/yr vs. $9,300,000/yr

Solving it in MATLAB

```
% we have 22 variable
% x1  x2  x3   x4   x5   x6   x7   x8   x9    x10   x11   x12   x13
% Qs  Qw  Rs1  Rs2  Rs3  R12  R13  R23  Qs11  Qs12  Qs13  Qs23  Q123
% x14   x15   x16   x17   x18   x19   x20   x21   x22
% Q213  Q223  Qs24  Q124  Q1w4  Q224  Q2w4  Q112  Q113
n=22;
% objective function
f=zeros(n,1);
f(1)=80000;
f(2)=20000;
% inequality constraints
A=[];
B=[];
% equality constraints
neq=17;
Aeq=zeros(neq,n);
Beq=zeros(neq,1);
Aeq(1,3)=1;     Aeq(1,9)=1;    Aeq(1,1)=-1;
Aeq(2,9)=1;                                                Beq(2)=30;
Aeq(3,6)=1;     Aeq(3,21)=1;                               Beq(3)=60;
Aeq(4,4)=1;     Aeq(4,3)=-1;   Aeq(4,10)=1;
Aeq(5,10)=1;    Aeq(5,21)=1;                               Beq(5)=90;
Aeq(6,7)=1;     Aeq(6,6)=-1;   Aeq(6,22)=1;  Aeq(6,13)=1;  Beq(6)=160;
Aeq(7,8)=1;     Aeq(7,14)=1;   Aeq(7,15)=1;                Beq(7)=320;
Aeq(8,5)=1;     Aeq(8,4)=-1;   Aeq(8,11)=1;  Aeq(8,12)=1;
Aeq(9,22)=1;    Aeq(9,14)=1;   Aeq(9,11)=1;                Beq(9)=240;
Aeq(10,13)=1;   Aeq(10,15)=1;  Aeq(10,12)=1;               Beq(10)=117;
Aeq(11,7)=-1;   Aeq(11,17)=1;  Aeq(11,18)=1;               Beq(11)=60;
Aeq(12,8)=-1;   Aeq(12,19)=1;  Aeq(12,20)=1;               Beq(12)=120;
Aeq(13,5)=-1;   Aeq(13,16)=1;
Aeq(14,17)=1;   Aeq(14,19)=1;  Aeq(14,16)=1;               Beq(14)=78;
Aeq(15,18)=1;   Aeq(15,20)=1;  Aeq(15,2)=-1;
Aeq(16,21)=1;    % which two cannot match
Aeq(17,22)=1;
% boundary
lb=zeros(n,1);
x=linprog(f,A,B,Aeq,Beq,lb)
```

TABLE 16.4 Expanded LP for Restricted Match in Example 16.3

Utility Cost:		$\min Z = 80000\,Q_S + 20000 Q_W$
Interval 1:	s.t.	$R_{S1} + Q_{S11} - Q_S = 0$
		$Q_{S11} = 30$
Interval 2:		$R_{12} + Q_{112} = 60$
		$R_{S2} - R_{S1} + Q_{S12} = 0$
		$Q_{S12} + Q_{112} = 90$
Interval 3:		$R_{13} - R_{12} + Q_{113} + Q_{123} = 160$
		$R_{23} + Q_{213} + Q_{223} = 320$
		$R_{S3} - R_{S2} + Q_{S13} + Q_{S23} = 0$
		$Q_{113} + Q_{213} + Q_{S13} = 240$
		$Q_{123} + Q_{223} + Q_{S23} = 117$
Interval 4:		$-R_{13} + Q_{124} + Q_{1W4} = 60$
		$-R_{23} + Q_{224} + Q_{2W4} = 120$
		$-R_{S3} + Q_{S24} = 0$
		$Q_{124} + Q_{224} + Q_{S24} = 78$
		$Q_{1W4} + Q_{2W4} - Q_W = 0$
Forbidden match:		$Q_{112} = Q_{113} = 0$ (H1–C1 do not exchange heat)

Same Q_s and Q_w, but not necessarily same for others

TABLE 16.4 Expanded LP for Restricted Match in Example 16.3

Utility Cost:	$\min Z = 80000\, Q_S + 20000 Q_W$
Interval 1:	s.t. $R_{S1} + Q_{S11} - Q_S = 0$
	$Q_{S11} = 30$
Interval 2:	$R_{12} + Q_{112} = 60$
	$R_{S2} - R_{S1} + Q_{S12} = 0$
	$Q_{S12} + Q_{112} = 90$
Interval 3:	$R_{13} - R_{12} + Q_{113} + Q_{123} = 160$
	$R_{23} + Q_{213} + Q_{223} = 320$
	$R_{S3} - R_{S2} + Q_{S13} + Q_{S23} = 0$
	$Q_{113} + Q_{213} + Q_{S13} = 240$
	$Q_{123} + Q_{223} + Q_{S23} = 117$
Interval 4:	$-R_{13} + Q_{124} + Q_{1W4} = 60$
	$-R_{23} + Q_{224} + Q_{2W4} = 120$
	$-R_{S3} + Q_{S24} = 0$
	$Q_{124} + Q_{224} + Q_{S24} = 78$
	$Q_{1W4} + Q_{2W4} - Q_W = 0$
Forbidden match:	$Q_{112} = Q_{113} = 0$ (H1–C1 do not exchange heat)

Solution of the problem gives us the minimum cost utility plus a realistic match of hot and cold streams!

Are we satisfied? What more can we include?

What is the minimum number of HEX?

Prediction of matches for minimum number of units

EXAMPLE 16.4

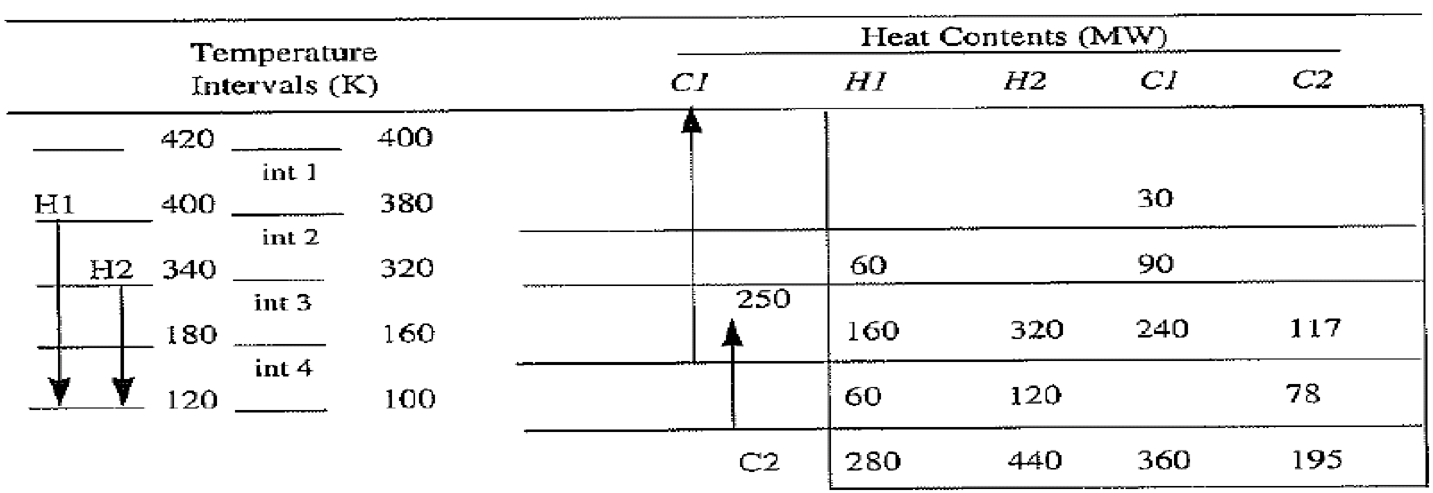
Let us consider again the problem in Table 16.1. We will assume that no constraints are imposed on the matches, so that 60 MW will be required for the heating and 225 MW for the cooling. Referring to Figure 16.8, which follows from Figure 16.4, Eqs. (16.12) to (16.14) lead to the problem shown in Table 16.5. If we solve the MILP, the solution that we obtain involves the six following matches:

Above pinch:

Match Steam–C1	60 MW	$(y_{S1A}=1,\ Q_{S11}=30,\ Q_{S12}=30)$
Match H1–C1	60 MW	$(y_{11A}=1,\ Q_{112}=60)$

Below pinch:

Match H1–C1	25 MW	$(y_{11B}=1,\ Q_{113}=25)$
Match H1–C2	195 MW	$(y_{12B}=1,\ Q_{123}=117,\ Q_{124}=78)$
Match H2–C1	215 MW	$(y_{21B}=1,\ Q_{123}=215)$
Match H2–W	225 MW	$(y_{2WB}=1,\ Q_{2W4}=225)$



Convert to the new detailed heat cascade diagram (Fig 16.8)

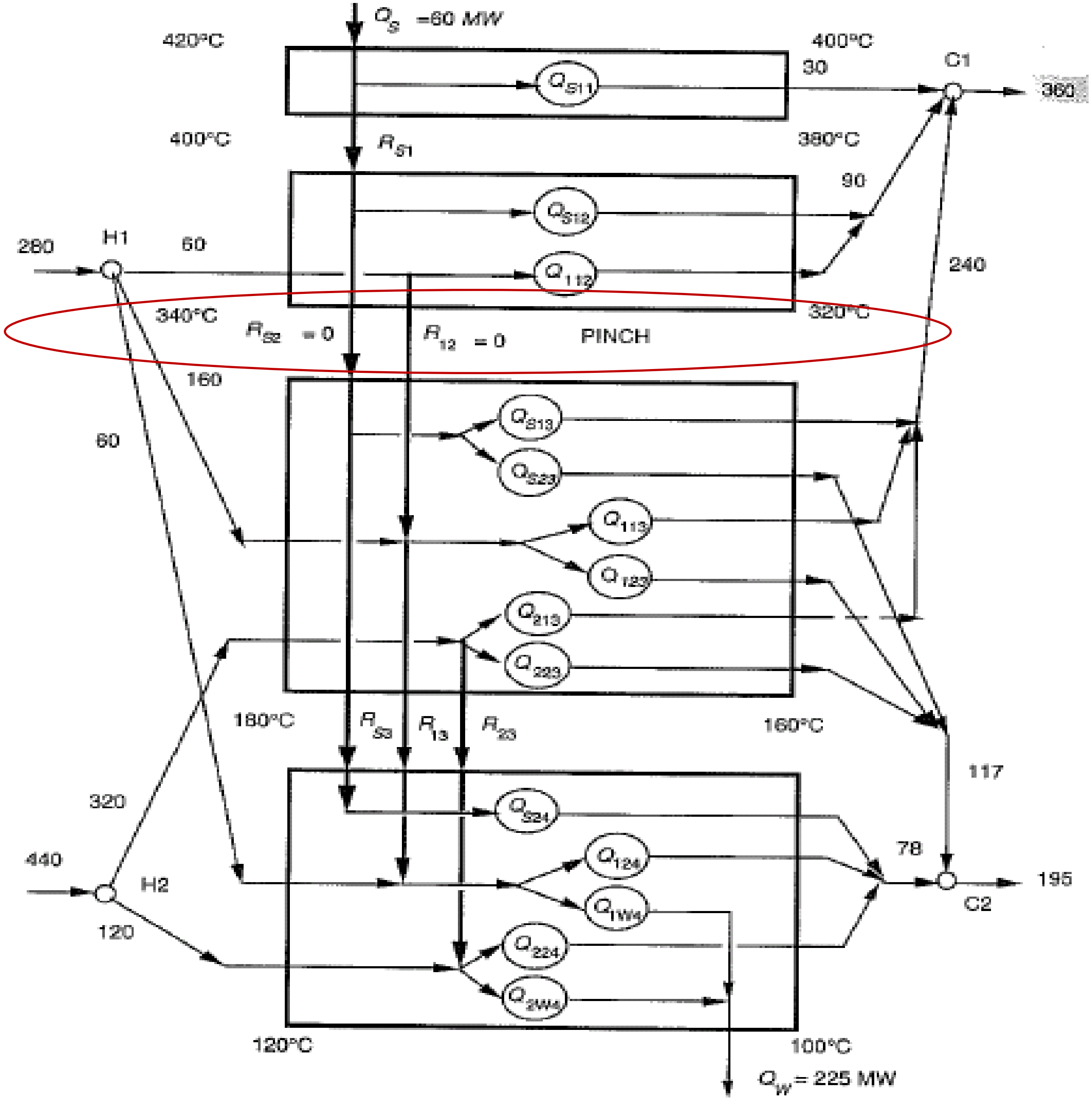


FIGURE 16.8 Representation of heat flows in MILP transshipment.

Derive the energy balance equations for each interval with the extended transhipment model

For match or not

$$\sum_{k=1}^{K^q} Q_{ijk} - U_{ij} y_{ij}^q \le 0$$

The upper bound U_{ij} is given by the smallest of the heat contents of the two streams

TABLE 16.5 MILP Model for Example 16.4

Number of units:		$\min Z = y_{S1}{}^A + y_{11}{}^A + y_{11}{}^B + y_{12}{}^B + y_{1W}{}^B + y_{21}{}^B + y_{22}{}^B + y_{2W}{}^B$
Interval 1:	s.t.	$R_{S1} + Q_{S11} = 60$ $Q_{S11} = 30$
Interval 2:		$R_{12} + Q_{112} = 60$ $R_{S2} - R_{S1} + Q_{S12} = 0$ $Q_{S12} + Q_{112} = 90$
Interval 3:		$R_{13} - R_{12} + Q_{113} + Q_{123} = 160$ $R_{23} + Q_{213} + Q_{223} = 320$ $Q_{113} + Q_{213} + Q_{S13} = 240$ $Q_{123} + Q_{223} + Q_{S23} = 117$
Interval 4:		$-R_{13} + Q_{124} + Q_{1W4} = 60$ $-R_{23} + Q_{224} + Q_{2W4} = 120$ $Q_{124} + Q_{224} + Q_{S24} = 78$ $Q_{1W4} + Q_{2W4} = 225$
Matches above pinch:		$Q_{S11} + Q_{S12} - 60\, y_{S1}{}^A \le 0$ $Q_{112} - 60\, y_{11}{}^A \le 0$
Matches below pinch:		$Q_{113} - 220\, y_{11}{}^B \le 0$ $Q_{123} + Q_{124} - 195\, y_{12}{}^B \le 0$ $Q_{1W4} - 220\, y_{1W}{}^B \le 0$ $Q_{213} - 240\, y_{21}{}^B \le 0$ $Q_{223} + Q_{224} - 60\, y_{22}{}^B \le 0$ $Q_{2W4} - 225\, y_{2W}{}^B \le 0$

```
% we have 27 variable
% x1    x2    x3    x4    x5    x6    x7    x8    x9   x10   x11   x12   x13   x14
% ys1A  y11A  y11B  y12B  y1wB  y21B  y22B  y2wB  Rs1  Qs11  R12   Q112  Rs2   Qs1
% x15   x16   x17   x18   x19   x20   x21   x22   x23   x24   x25   x26
% R13   Q113  Q123  R23   Q213  Qs13  Q223  Qs23  Q124  Q224  Qs24  Q1w4
n=27;
% objective function
f=zeros(n,1);
% 8 binary
nb=8;
f(1:nb)=1;
% inequality constraints
nie=8;
A=zeros(nie,n);
B=zeros(nie,1);
A(1,10)=1;   A(1,14)=1;    A(1,1)=-60;
A(2,12)=1;                 A(2,2)=-60;
A(3,16)=1;                 A(3,3)=-220;
A(4,17)=1;   A(4,23)=1;    A(4,4)=-195;
A(5,26)=1;                 A(5,5)=-220;
A(6,19)=1;                 A(6,6)=-240;
A(7,21)=1;   A(7,24)=1;    A(7,7)=-195;
A(8,27)=1;                 A(8,8)=-225;
% equality constraints
neq=13;
Aeq=zeros(neq,n);
Beq=zeros(neq,1);
Aeq(1,9)=1;    Aeq(1,10)=1;                                Beq(1)=60;
Aeq(2,10)=1;                                               Beq(2)=30;
Aeq(3,11)=1;   Aeq(3,12)=1;                                Beq(3)=60;
Aeq(4,13)=1;   Aeq(4,9)=-1;  Aeq(4,14)=1;                  Beq(4)=0;
Aeq(5,14)=1;   Aeq(5,12)=1;                                Beq(5)=90;
Aeq(6,15)=1;   Aeq(6,11)=-1; Aeq(6,16)=1;  Aeq(6,17)=1;    Beq(6)=160;
Aeq(7,18)=1;   Aeq(7,19)=1;  Aeq(7,21)=1;                  Beq(7)=320;
Aeq(8,16)=1;   Aeq(8,19)=1;  Aeq(8,20)=1;                  Beq(8)=240;
Aeq(9,17)=1;   Aeq(9,21)=1;  Aeq(9,22)=1;                  Beq(9)=117;
Aeq(10,15)=-1; Aeq(10,23)=1; Aeq(10,26)=1;                 Beq(10)=60;
Aeq(11,18)=-1; Aeq(11,24)=1; Aeq(11,27)=1;                 Beq(11)=120;
Aeq(12,23)=-1; Aeq(12,24)=1; Aeq(12,25)=1;                 Beq(12)=78;
Aeq(13,26)=1;  Aeq(13,27)=1;                               Beq(13)=225;
% boundary
lb=zeros(n,1);
ub=ones(n,1)*Inf;
ub(1:nb)=1;
% binary is integer
intcon=[1:1:nb];
x=intlinprog(f,intcon,A,B,Aeq,Beq,lb,ub)
```

TABLE 16.5 MILP Model for Example 16.4

Number of units: $\min Z = y_{S1}{}^A + y_{11}{}^A + y_{11}{}^B + y_{12}{}^B + y_{1W}{}^B + y_{21}{}^B + y_{22}{}^B + y_{2W}{}^B$

Interval 1: s.t.

$$R_{S1} + Q_{S11} = 60$$
$$Q_{S11} = 30$$

Interval 2:

$$R_{12} + Q_{112} = 60$$
$$R_{S2} - R_{S1} + Q_{S12} = 0$$
$$Q_{S12} + Q_{112} = 90$$

Interval 3:

$$R_{13} - R_{12} + Q_{113} + Q_{123} = 160$$
$$R_{23} + Q_{213} + Q_{223} = 320$$
$$Q_{113} + Q_{213} + Q_{S13} = 240$$
$$Q_{123} + Q_{223} + Q_{S23} = 117$$

Interval 4:

$$-R_{13} + Q_{124} + Q_{1W4} = 60$$
$$-R_{23} + Q_{224} + Q_{2W4} = 120$$
$$Q_{124} + Q_{224} + Q_{S24} = 78$$
$$Q_{1W4} + Q_{2W4} = 225$$

Matches above pinch:

$$Q_{S11} + Q_{S12} - 60\, y_{S1}{}^A \le 0$$
$$Q_{112} - 60\, y_{11}{}^A \le 0$$

Matches below pinch:

$$Q_{113} - 220\, y_{11}{}^B \le 0$$
$$Q_{123} + Q_{124} - 195\, y_{12}{}^B \le 0$$
$$Q_{1W4} - 220\, y_{1W}{}^B \le 0$$
$$Q_{213} - 240\, y_{21}{}^B \le 0$$
$$Q_{223} + Q_{224} - 60\, y_{22}{}^B \le 0$$
$$Q_{2W4} - 225\, y_{2W}{}^B \le 0$$

Above pinch:

Match Steam–C1	60 MW	($y_{S1A} = 1$, $Q_{S11} = 30$, $Q_{S12} = 30$)
Match H1–C1	60 MW	($y_{11A} = 1$, $Q_{112} = 60$)

Below pinch:

Match H1–C1	25 MW	($y_{11B} = 1$, $Q_{113} = 25$)
Match H1–C2	195 MW	($y_{12B} = 1$, $Q_{123} = 117$, $Q_{124} = 78$)
Match H2–C1	215 MW	($y_{21B} = 1$, $Q_{123} = 215$)
Match H2–W	225 MW	($y_{2WB} = 1$, $Q_{2W4} = 225$)

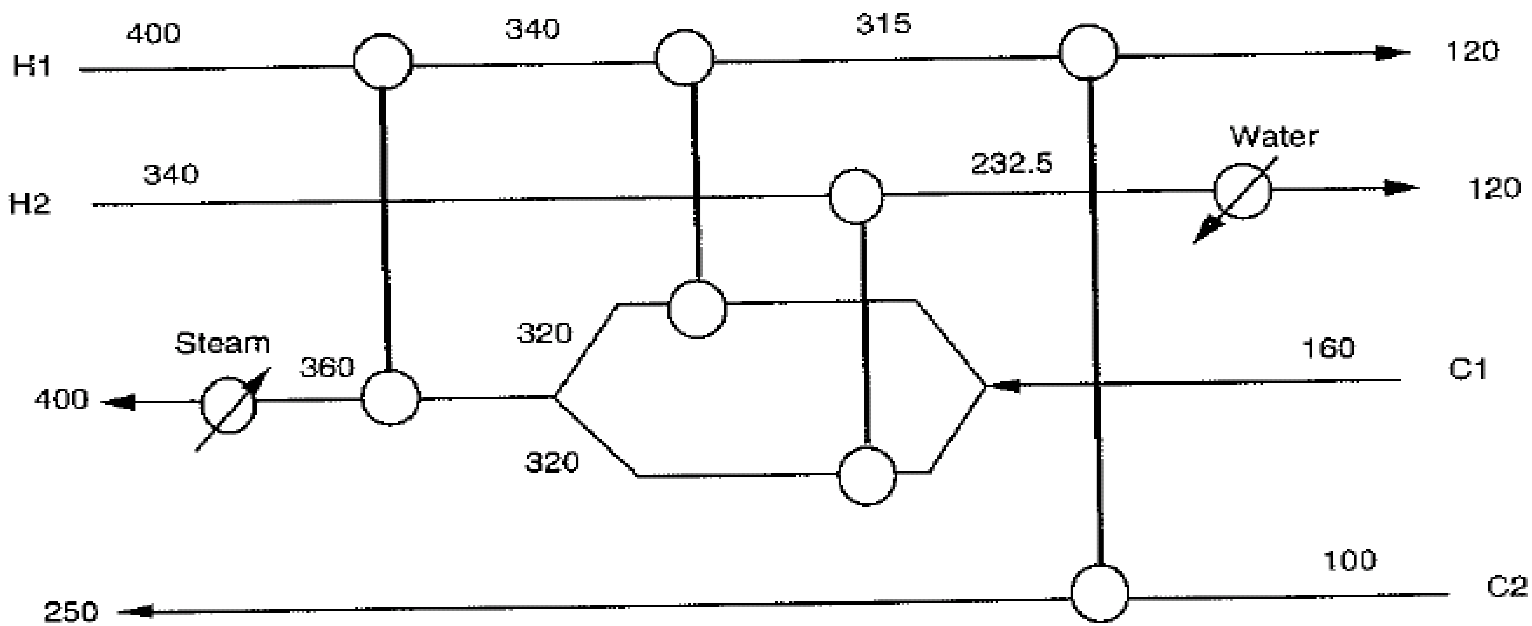


FIGURE 16.9 Network configuration for matches predicted from MILP in Example 16.4.

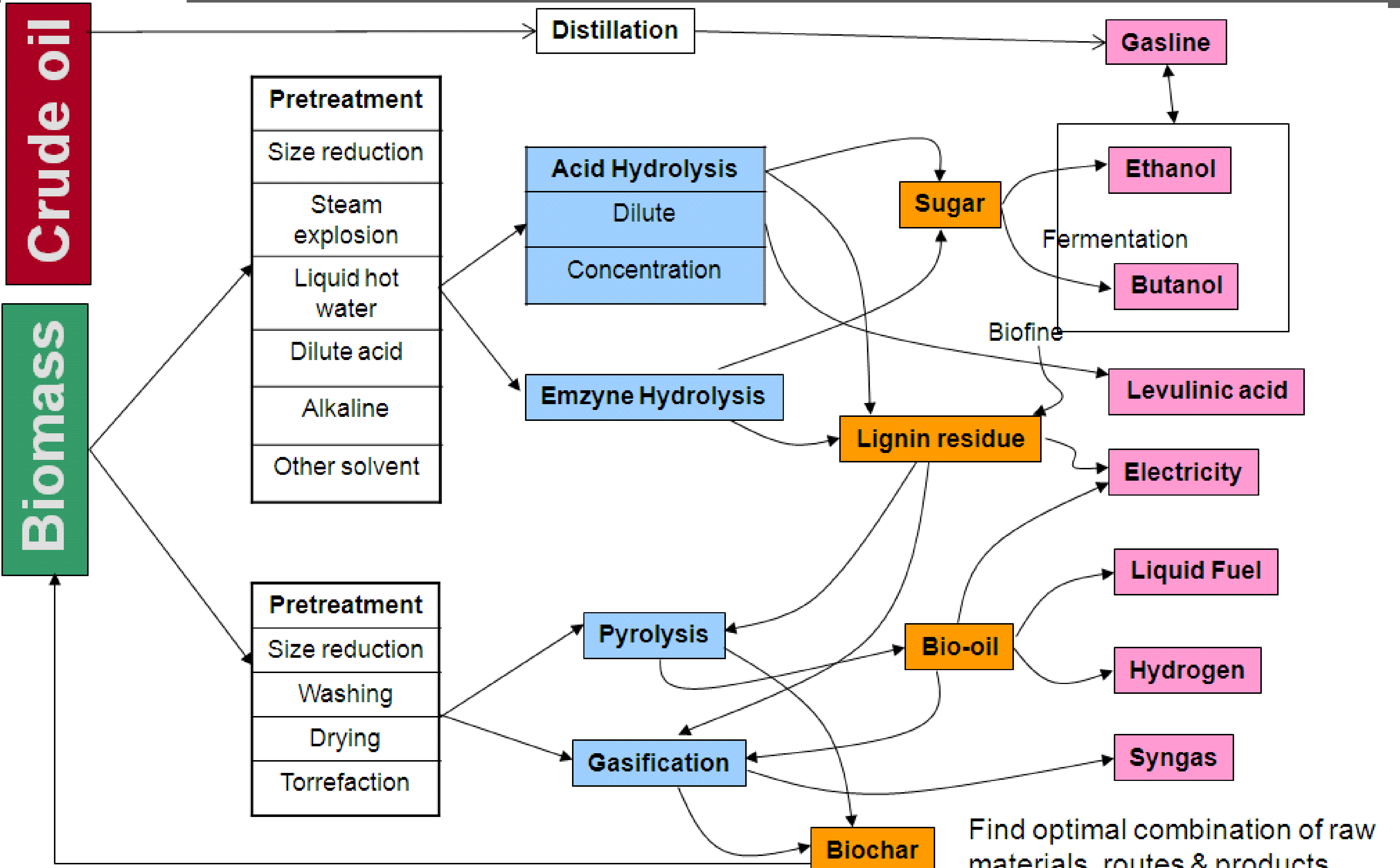
What Next?

Add heat exchanger design equations ($Q = UA\Delta T_{lm}$)
Add heat exchanger cost equations (cost = f(A))

Result: MINLP Transhipment model (simultaneous optimization strategy) – rest of chapter 16.

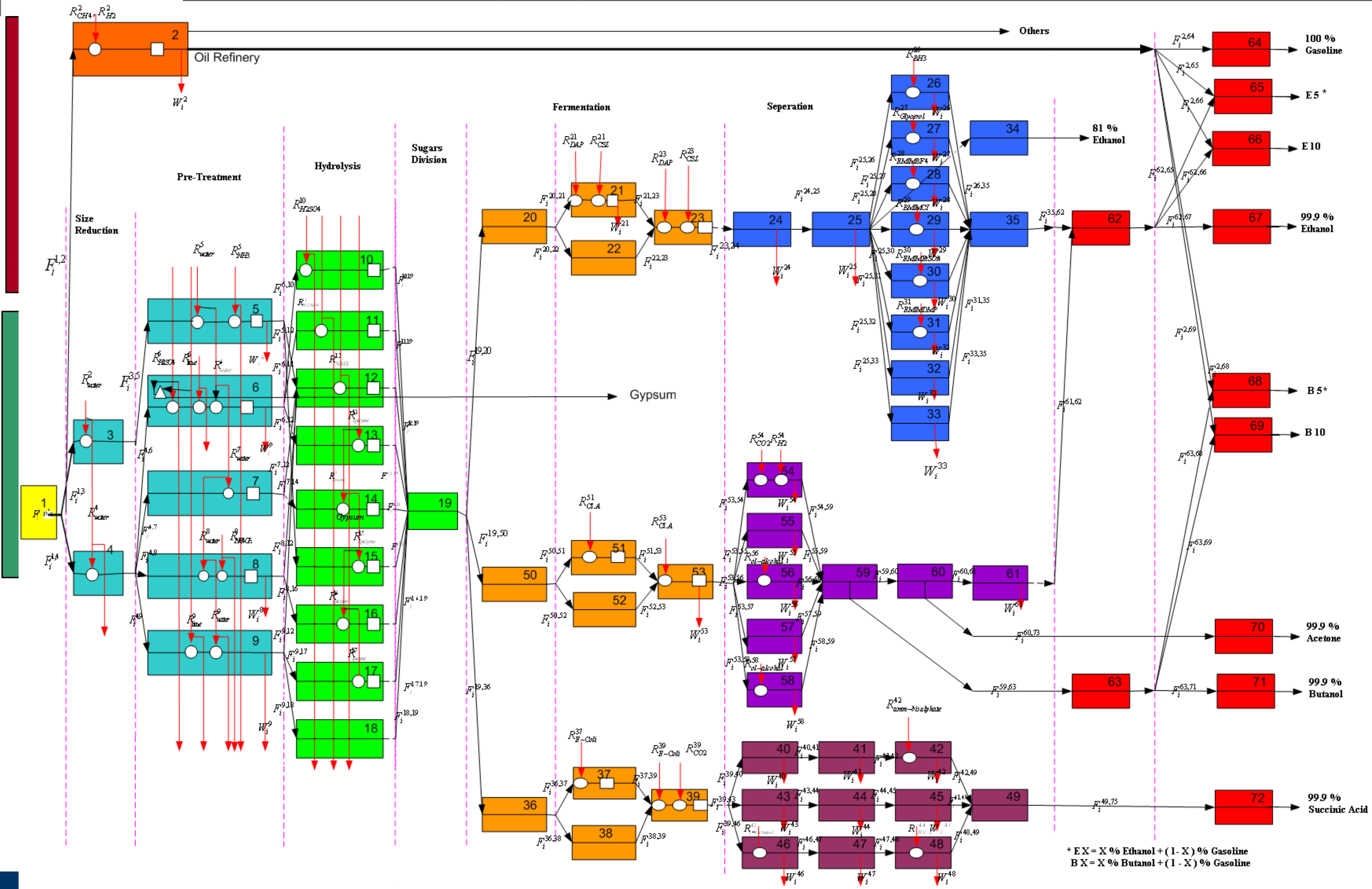
Note: MINLP = Mixed Integer Non-Linear Programming

Research Highlights – I: Biorefinery data

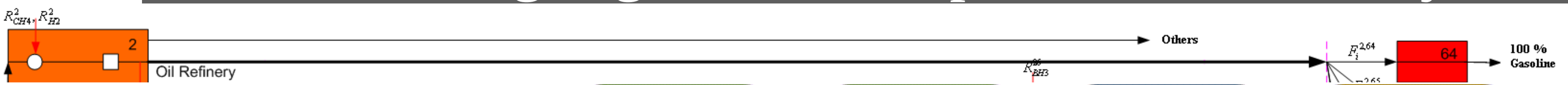


Find optimal combination of raw materials, routes & products

Research Highlights – II: Superstructure



Research Highlights – III: Optimal Biorefinery

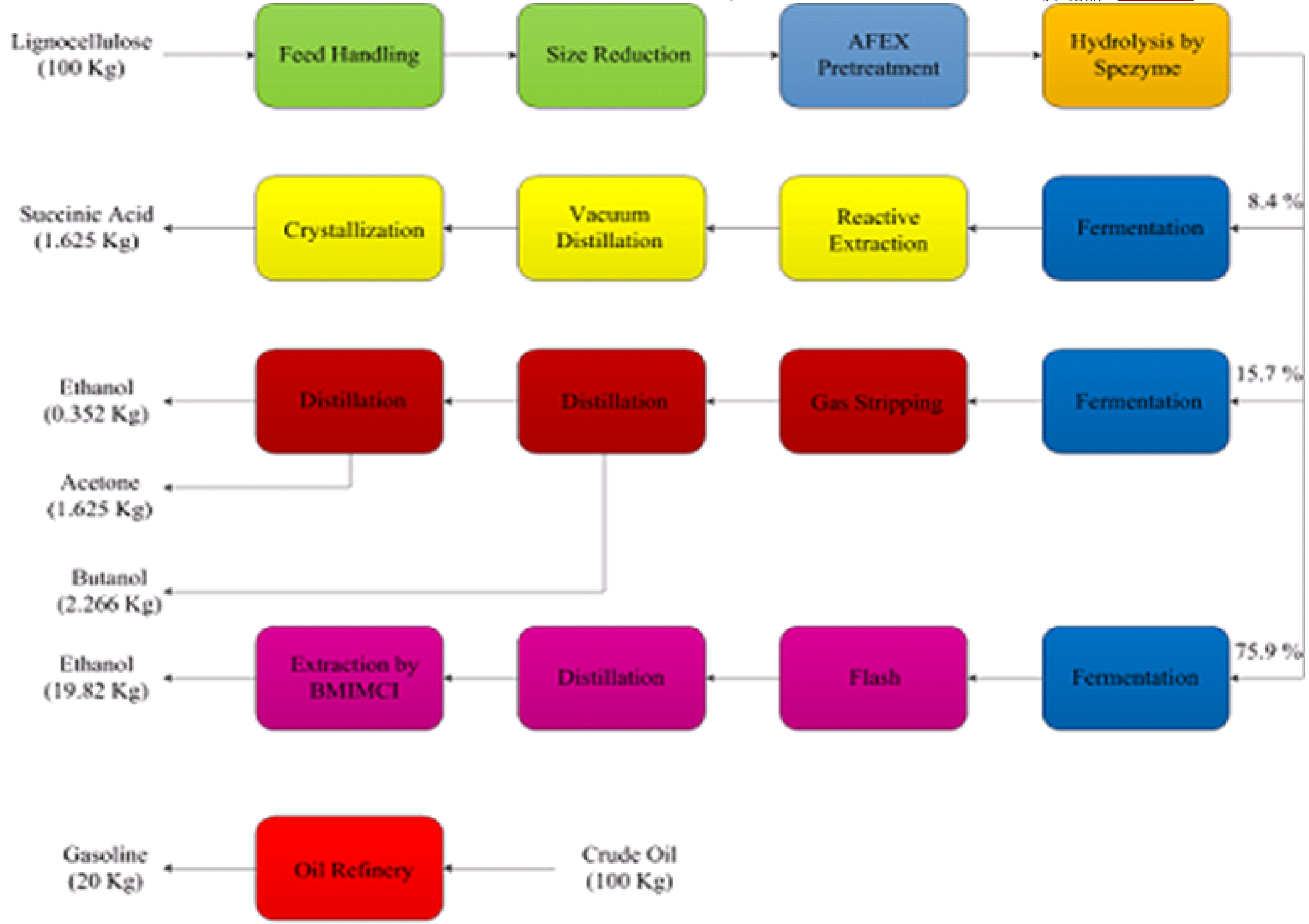


7 stages

72 processing steps

452179 equations

68 binary decisions



Fobj = f(products, raw material, chemicals, waste, fixed cost) = 35.3 USD/100kg of biomass

Manage the complexity by decomposition: example

Objective function

$$\min 2x_1 + 3x_2 + 1.5y_1 + 2y_2 - 0.5y_3 \quad \text{IV} \quad (1)$$

$s.t.$

Process model

$$x_1^2 + y_1 = 1.25 \quad (2)$$

$$x_2^{1.5} + 1.5y_2 = 3.0 \quad (3)$$

II

Process constraints

$$x_1 + y_1 \le 1.60 \quad (4)$$

$$1.333x_2 + y_2 \le 3.00 \quad (5)$$

III

Flowsheet constraints

$$-y_1 - y_2 + y_3 \le 0 \quad (6)$$

$$y_1 y_2 = 1 \quad (7)$$

I

Variable bounds

$$x_1, x_2 \ge 0 \quad (8)$$

$$y_1, y_2, y_3 = \{0,1\} \quad (9)$$

Solution strategy:

Solve I: Y1 = 1, Y2 = 1, Y3 =0; Y1= 1, Y2= 1, Y3 = 1 (only two feasible sets)

Solve II: X1 = 0.5; X2 = 0.544 (for both sets of Y)

Solve III: Eq. 4 & Eq. 5 are satisfied for both sets of Y and the calculated values of X

Solve IV: Eq 1 = 6.132 for set 1; = 5.632 for set 2

Global optimal solution: set 2 (X1=0.5, X2=0.544, Y1=1, Y2=1, Y3=1)